MANAGING INFORMATION COLLECTION IN SIMULATION-BASED DESIGN

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MANAGING INFORMATION COLLECTION IN SIMULATION-BASED DESIGN

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To my grandmother for the endless bowls of cream of wheat.

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The completion of this thesis was more challenging then I could have imagined. I could not have done it without the support of my colleagues, friends, and family.

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α	the optimism-pessimism index for the Hurwicz decision criterion
a	a possible action
$A=\{a\}$	the set of all possible actions
a^*	the optimal action, $a^* = \arg\max_{a} \; (E_{\tilde{p}(x)}[\pi(a,x)])$
a_0^*	the optimal action prior to receiving message y
a_y^*	the optimal action with information y
d_0	the decision made before using the more accurate model
d_1	the decision made after using the more accurate model
ε	the level of accuracy of a model
E_x	an expectation taken over a random variable, X
$E_x[\pi(a,x)]$	actual expected payoff
$E_{\tilde{p}(x)}[\pi(a,x)]$	expected payoff calculated using $\tilde{p}(x)$
$f(x)$	the output of a model
$\mathbf{I}$	an information source
$N(\mu,\sigma^2)$	a normal distribution with mean, μ, and variance, σ^2
π	payoff
$\pi^*_{\tilde{p}(x)}$	estimated expected payoff according to $\tilde{p}(x)$
π_{true}	true expected payoff
$p(x)$	a probability density function
$\tilde{p}(x)$	a subjective probability density function

$\tilde{p}(x \mid y)$	a posterior probability distribution function
$P(H)$	the probability of event H
σ^2	variance of a random variable
$[\underline{\sigma}^2, \bar{\sigma}^2]$	bounds on the variance of a random variable
s	standard deviation of a random variable
μ	mean of a random variable
$\hat{\mu}$	estimate of the mean of a random variable
$[\underline{\mu}, \bar{\mu}]$	bounds on the mean of a random variable
U	utility of some action
$\underline{U}$	lower bound on utility
$\bar{U}$	upper bound on utility
U^{α}	decision point based on the Hurwicz decision criterion $U^{\alpha} = \alpha\underline{U} + (1-\alpha)\bar{U}$
$U^{\alpha=0.5}$	utility predicted by the $\alpha = 0.5$ Hurwicz decision criterion
$[\underline{U}_i, \bar{U}_i]_{i=1..m}$	m possible utility intervals for a more accurate model
$U_1'^{\alpha}$, $U_0'^{\alpha}$	hypothetical decision points
$\underline{U}'$, $\bar{U}'$, $\underline{U}''$, $\bar{U}''$	hypothetical bounds on utility
$\bar{U}_{\max}$	greatest possible utility given current knowledge
$\upsilon(y)$	gross value of a message y
$\upsilon(y \mid x)$	ex-post gross value of the message y
$V(n+1 \mid \Sigma)$	average value of the next sample
$\tilde{V}(n+1)$	approximate interval of the value of the next sample
$[\underline{V}, \bar{V}]$	bounds on the value of a more accurate model

$\left[\underline{V}_{net}, \overline{V}_{net}\right]$	bounds on net value of a more accurate model
$\left[\underline{V}_{p}, \overline{V}_{p}\right]$	bounds on the value of a perfect model
$\left[\underline{V}_{post}, \overline{V}_{post}\right]$	bounds on the net value of a more accurate model
x	a realization of X
$\{x_i\}_{i=1}^{n}$	a set of n realizations of the random variable X
X	a random variable
y	a message from an information source
$\mathbf{Y}$	set of all possible messages from an information source

Summary

An important element of successful engineering design is the effective management of resources to support design decisions. Design decisions can be thought of as having two phases—a formulation phase and a solution phase. As part of the formulation phase, engineers must decide how much information to collect and which models to use to support the design decision. Since more information and more accurate models come at a greater cost, a cost-benefit trade-off must be made. Previous work has considered such trade-offs in decision problems when all aspects of the decision problem can be represented using precise probabilities, an assumption that is not justified when information is sparse.

In this thesis, we use imprecise probabilities to manage the information cost-benefit trade-off for two decision problems in which the quality of the information is imprecise: 1) The decision of when to stop collecting statistical data about a quantity that is characterized by a probability distribution with unknown parameters; and 2) The selection of the most preferred model to help guide a particular design decision when the model accuracy is characterized as an interval. For each case, a separate novel approach is developed in which the principles of information economics are incorporated into the information management decision.

The problem of statistical data collection is explored with a pressure vessel design. This design problem requires the characterization of the probability distribution that describes

a novel material's strength. The model selection approach is explored with the design of an I-beam structure. The designer must decide how accurate of a model to use to predict the maximum deflection in the span of the structure. For both problems, it is concluded that the information economic approach developed in this thesis can assist engineers in their information management decisions.

1 Introduction

Engineering design is a sequential and iterative process, consisting of five phases: product planning, clarification of task, conceptual design, embodiment design, and detail design (Pahl and Beitz 1996). Decision making is an important part of each of these phases and is formalized in decision-based design research (Thurston 1990; Hazelrigg 1996; Marston, Allen et al. 2000). Figure 1 shows a flowchart of the decision process (Clemen 1996). At the beginning of the decision, the decision maker (DM) identifies the decision situation and his or her objectives. The solution alternatives are then identified and the decision problem is decomposed and modeled so that it can be solved in a systematic way. Based on the decomposition and modeling, a best alternative is chosen and some sensitivity analysis is performed to test how sensitive the choice of the best alternative is to the decision problem parameters and the decomposition and modeling methods. Using the results from the sensitivity analysis, it is decided that either the solution is satisfactory or that iteration is required with a modified decision problem.

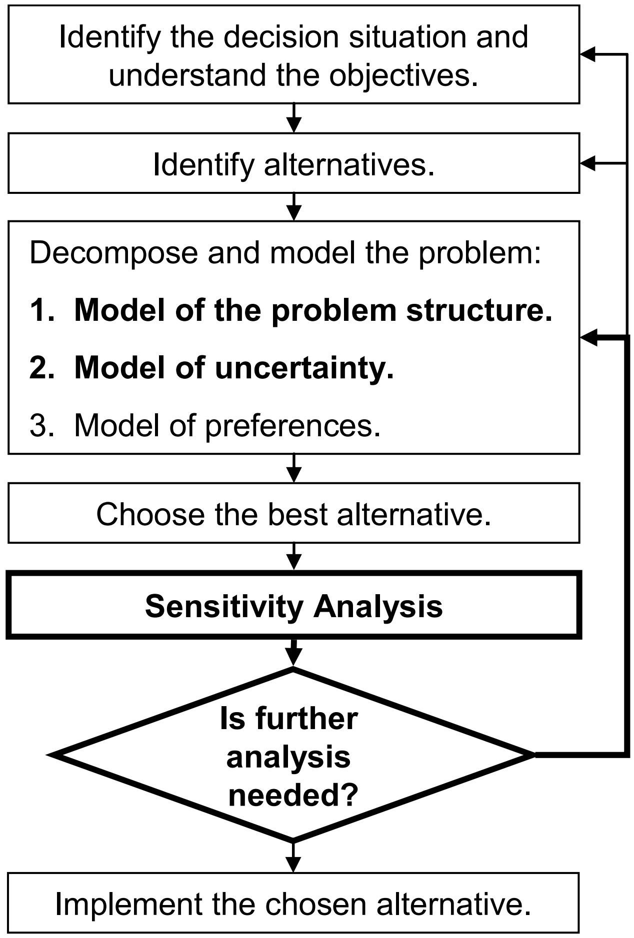

Figure 1: A decision process flowchart, adapted from (Clemen 1996)

This thesis focuses the elements that are bolded in Figure 1. Specifically, it addresses the sub-decision problems regarding what information to gather (updating the model of uncertainty) and which models to use to guide the decision (selecting a model of the problem structure).

1.1 Motivation

Although better information and models may provide value to the designers by leading to a better final design, whether an information source is valuable cannot be known with certainty. Until resources are spent acquiring information or developing models, the exact information obtained from the information source is unknown and its value is therefore uncertain. This thesis presents an approach for integrating the management of this cost-benefit tradeoff into the design decision model using the principles defined as *information economics* (Marschak 1974). *Economics* is the *study of choice under conditions of scarcity* (Lieberman and Hall 2000). Extending this definition, *information economics* is the study of choice in information collection and management when resources, such as time and money, to expend on information are scarce.

Motivating Question: How should a decision maker decide on resource allocations when gathering information in support of design decisions?
Hypothesis: The principles of information economics can guide resource allocation by predicting bounds on the value of gathering additional information and of developing better models.

In current engineering practice, the principles of information economics are rarely used to guide information collection decisions. Four examples common in engineering practice are provided below, each of which illustrates the lack of an information economic perspective.

Statistical data collection: When collecting statistical data, a DM will often specify a confidence level for which he or she wishes to be sure that one design alternative outperforms another. Given an initial set of collected data, an estimate of the number of data samples required to conclude the superiority of one alternative over another at the specified confidence level can be derived using statistical principles based on the assumption that the true distribution is Gaussian. This approach has two flaws: 1) The DM has no systematic guidance in specifying an appropriate confidence level; usually a confidence level of 90%, 95%, or 99% is specified, often without much analysis of the design problem. 2) The required number of additional data samples is independent of the cost of data collection. It may be prohibitively costly to collect the required amount of additional data. Without information economic principles, the DM is left with little guidance for information collection decisions in such cases.

Monte Carlo analysis: When performing a Monte Carlo analysis of a discrete event simulation, the DM uses an approach similar to that for statistical data collection to determine the number of simulation replicates required such that a hypothesis can be verified at a specified confidence level. Again, the cost of collecting data—in this case performing simulations—is not taken into account. If the simulation is very complex, the cost of performing the simulations can be a major factor in the DM's choice of the number of replicates. Information economics provides a framework to manage explicitly the trade-off between the more accurate knowledge attained by performing additional simulation replicates and the cost of performing such simulations.

Model selection: In general, the DM has little systematic guidance in model selection. To compensate for this lack of guidance, he or she will often be conservative and choose to acquire and use a model that is more accurate then is really required, spending additional resources unnecessarily. In addition, each design decision may benefit most from a different model. The choice of the best model in support of a given design decision depends on the preference of the DM, the uncertainty in other problem parameters, the importance (in dollars) of the decision, etc. Clearly, a framework that allows the DM to trade-off explicitly the value derived from using more accurate models with the cost of such models would be useful.

Finite element models: A more structured model selection decision faced by the DM is the choice of the number of elements to use in a finite element model. In current practice, the DM studies the convergence of the finite element model as the number of elements is increased. The DM then selects a number of elements that will yield a result within a specified error tolerance. But such an approach does not factor in the cost of running the selected model, which can be significant for complex models. Information economics provides a framework for which the DM's accuracy preferences and the cost of running the model can be traded off explicitly.

This thesis addresses two of these information collection decisions. Specifically, this thesis develops and presents information economic approaches for decisions about statistical data collection and model selection. These approaches are explored on a theoretical level and are applied to design problems to build confidence in their usefulness.

1.1.1 Research question 1 and hypothesis

DMs often face the decision of when to stop collecting statistical data during the process of characterizing a probability distribution that describes a random process. For example, a DM may want to build a concrete structure out of a new type of concrete mix. To decide whether or not this new concrete mix should be used, its performance is tested. It is assumed that the yield strength is well modeled as a normally distributed random variable, but that the parameters of the normal distribution are unknown. The DM can estimate the parameters by testing samples to failure and recording the failure stress of each. But how many samples are needed to make an accurate determination of the new mix's strength distribution? Certainly, one is too few and one million would probably be too many, but how can the DM decide how many specimen should be tested? Is there a correct number?

Research Question 1: How should a decision maker decide when to stop gathering statistical data when trying to characterize a probability distribution describing a random event?
Hypothesis: The principles of information economics allow the DM to bound the value of the next statistical data point and these bounds can guide the DM towards better decisions about statistical data collection.

Taking an information economic perspective, we come to the realization that once a certain amount of data has been collected, the cost of gathering additional samples becomes greater than the benefit that the additional samples provide. Using information economic principles, the DM can monitor the cost and benefit of additional data collection throughout the data collection process and identify the point when the benefit is outweighed by the cost. In general, the DM should collect data samples until this point is reached. Information economic principles allow the DM to explicitly manage the trade-off between the accuracy of his or her knowledge and the cost of data collection.

Necessary background knowledge for addressing research question 1 is provided in Chapter 2. The derivation of an information economic approach to statistical data collection and an application is presented in Chapter 3.

1.1.2 Research question 2 and hypothesis

In many design problems, the DM must decide which of several engineering models to develop to support a decision. More accurate models yield more accurate knowledge but at a greater cost—another trade-off. If given the choice of several models, which one should the DM select? Is there a correct choice?

Research Question 2: How should a decision maker select the most preferred model given a particular design problem?
Hypothesis: The principles of information economics allow the value of more accurate models to be bounded. The DM can use such bounds to guide model selection decisions.

An information economic approach allows the DM to systematically predict the amount of value that different models would contribute to the particular design decision based on the model's accuracy. With such information, the DM can explicitly manage the cost-benefit trade-off between model accuracy and model cost, allowing the most preferred model to be selected.

Necessary background knowledge for addressing research question 2 is provided in Chapter 2. In Chapter 4, an approach for bounding the value of more accurate models using information economics is derived and an application is presented.

1.2 Organization of Thesis

The organization of the thesis is illustrated in Figure 2. Chapter 1 provides motivation for the problem and highlights the research questions to be answered. Chapter 2 overviews relevant knowledge, literature, and previous work. Chapter 3 addresses research question 1 by deriving an information economic approach to decisions about statistical data collection and explores the approach with an example pressure vessel design. Chapter 4 addresses research question 2 and explores a derived information economic approach to model selection in the context of the design of an I-beam structure. Chapter 5 summarizes the contributions of this thesis, examines limitations, and identifies possible extensions.

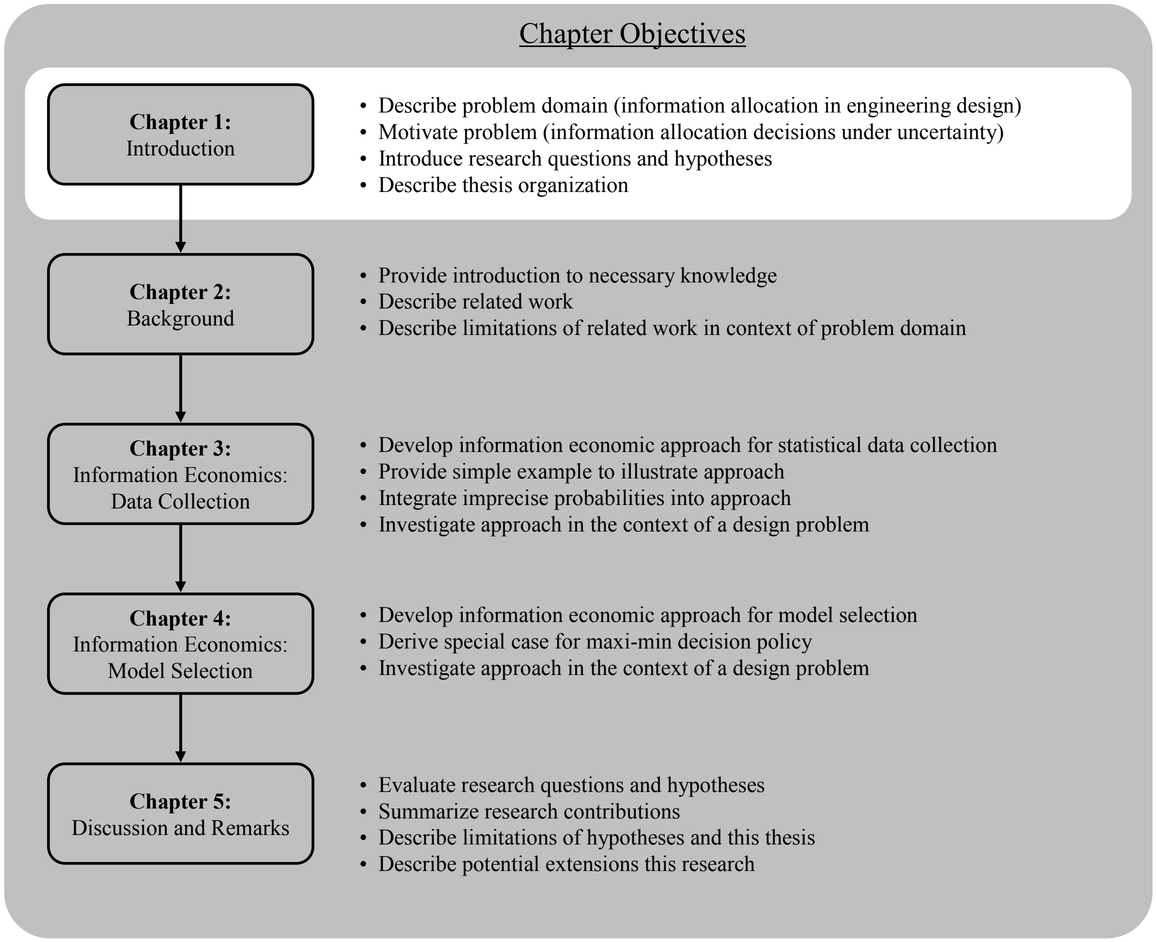

Figure 2: The organization of this thesis

2 Background

This chapter provides an overview of topics that are foundational to this thesis and describes related research from the engineering community. The purpose of this chapter is to familiarize the reader with the background knowledge necessary to better understand the remainder of this thesis and to bring to light some of the limitations that exist in the literature.

The chapter begins with an overview of imprecise probabilities followed by an explanation of payoff and utility functions, functions used by the designer to express preferences, i.e., judge the success of design alternatives. By integrating imprecise probabilities and the designer's utility function, we arrive at imprecise utility functions an imprecise characterization of preferences that is no longer transitive. To make decisions based on imprecise utility functions, decision policies under imprecision are required, the next topic in the chapter. The chapter then describes the principles of information economics and explains how we can think of the information cost-benefit trade-offs in engineering design in relation to such principles. The chapter concludes with a summary of related research from the engineering design community and identifies the knowledge gap that is addressed in this thesis. Additional background information is provided throughout the thesis where needed. A broader investigation of information economics and uncertainty in engineering design can be found in the Ph.D. dissertation of Jason Aughenbaugh (Aughenbaugh 2006).

2.1 Imprecise probabilities

Uncertainty is often represented using probabilities. From among the many possible interpretations of probability (Savage 1972; de Finetti 1980; Walley 1991; Joslyn and Booker 2005), we use a subjective interpretation of probability. We avoid a *frequentist* interpretation, under which a probability represents the ratio of times that one outcome occurs compared to the total number of outcomes in a series of identical, repeatable, and possibly random trials. While there may be random variables that assume outcomes according to true relative frequencies, we choose the subjective interpretation because the true relative frequencies cannot be determined with any finite number of data samples, and because a subjective interpretation is applicable to a broader class of problems, as it is not limited to repeatable events. Naturally, subjective probabilities should be consistent with available information, including knowledge about observed relative frequencies and the DM's actual beliefs; such probabilities can be considered *rationalist* subjective probabilities (Walley 1991).

Under a *subjective* interpretation, probabilities are an expression of belief based on an individual's willingness to bet (de Finetti 1980; Winkler 1996; Joslyn and Booker 2005). Every bet has a price associated with it, and one can either buy or sell a bet at that price. The use of precise probabilities presumes that a DM can determine exactly the price at which he or she is indifferent between buying and selling the bet, the DM's so-called fair price (de Finetti 1974).

The use of *imprecise probabilities* allows for a range of prices at which a DM would neither buy nor sell the bet, because he or she is not sure how betting at these prices will affect his or her expected payoff. For instance, consider a bent quarter. The DM is uncertain about whether it will land heads-up or tails-up on a given toss; and until he or she has seen it flipped many times, he or she is also uncertain about how probable it is that it will land heads-up or tails-up. However, the DM is confident that the probability of the bent quarter landing heads-up is greater than 0.3 and less than 0.6. The DM can state such a belief using imprecise probabilities as $P(H)=[0.3, 0.6]$. If a bet was established that paid \$1 for the outcome of heads-up, then the DM would buy this bet at any amount less then \$0.30 and sell it at any amount greater then \$0.60. The DM would neither buy nor sell the bet for any amount between \$0.30 and \$0.60 because he or she would be unsure as to how such bets would affect his or her expected payoff.

In theory, imprecise probabilities can be reduced to precise probabilities by collecting infinite evidence and expending infinite effort to elicit the DM's beliefs. In the example problems, we are explicitly assuming a finite amount of evidence, such that precise probabilities are unattainable. We therefore use imprecise probabilities to capture the DM's current state of information.

Imprecise probabilities have been formalized by Walley (Walley 1991), and the value of using imprecise probabilities in certain engineering design decisions has been demonstrated previously (Aughenbaugh and Paredis 2005). We extend this work to estimate the value of information through the application of information economics and imprecise probabilities. Imprecise probabilities allow for the value of future information to be predicted as explained in Chapter 3. Other common representations of imprecision in probabilities can be found in the multi-attribute decision-making literature including ordinal ranking of probabilities (Sarin 1978; Weber 1987) and probabilities subject to linear constraints (Kmietowicz and Pearman 1984).

2.2 The probability-box

In this thesis, we use a probability-box or *p-box* (Ferson and Donald 1998), to represent imprecise probabilities. A p-box incorporates both imprecision and probabilistic characterizations by expressing *interval* bounds on the cumulative *probability* distribution function (CDF) for a random variable. More formally, the bounds on a p-box, such as shown in Figure 3(a), are given by two CDFs (F_1 and F_2) that enclose a set of CDFs that are consistent with the current state of information and the DM's beliefs. The p-box shown in Figure 3(a) is for a random variable Z with known variance $\sigma^2 = 1$ and mean bounded by the interval $\mu = [0,1]$. Thus extending the notation of probability, we can write $Z \sim N([0,1],1)$.

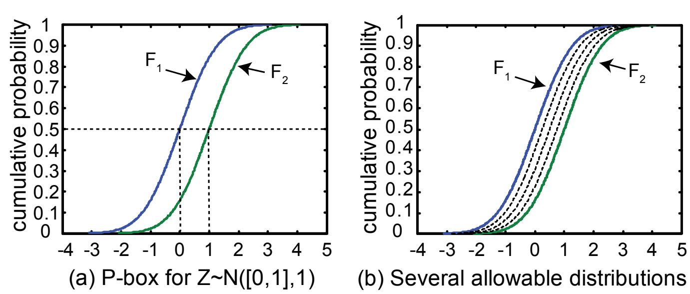

Figure 3: Example P-box and distributions

The true CDF is unknown, and any of the infinite number of normal CDFs with $\sigma^2 = 1$ inside the p-box could be the true one, such as those shown in Figure 3(b). Although p-boxes are not restricted to characterizing normal distributions, we limit our discussion to this case in the interest of clarity.

Only recently have researchers addressed the construction of p-boxes from sample data (Ferson, Hajagos et al. 2005). While several approaches exist (Ferson, Hajagos et al. 2005), we choose a pragmatic approach based on standard statistical confidence intervals. In this thesis, we assume the random variable (X) is normally distributed, but with unknown mean and variance:

$$X \sim Normal(\mu, \sigma^2). \tag{1}$$

One basis of reference for the true but unknown μ and σ^2 are the minimum variance unbiased point estimates:

$$\hat{\mu} = \overline{x} = \tfrac{1}{n}\sum_{i=1}^{n} x_i \tag{2}$$

$$\hat{\sigma}^2 = s^2 = \tfrac{1}{n-1}\sum_{i=1}^{n} (x_i - \overline{x})^2 \tag{3}$$

where the x_i's are realizations of the random variable, and n is the sample size. These quantities are called respectively the sample mean and sample variance and are commonly used in pure probabilistic approaches. In order to construct a p-box, we broaden these point estimates to confidence intervals. In this experiment, a 95% confidence level is used.

2.2.1 Confidence interval for the mean

Since x_1, x_2, …, x_n is a random sample from a normal distribution with unknown mean μ and unknown variance σ^2, the sampling distribution of the statistic

$$t = \frac{\hat{\mu} - \mu}{s/\sqrt{n}} \tag{4}$$

is the *t* distribution with $n-1$ degrees of freedom. Letting $t_{\alpha/2,n-1}$ be the upper $\alpha/2$ percentage point of the *t* distribution with $n-1$ degrees of freedom, it can be shown that

$$P\left\{-t_{\alpha/2,n-1} \le t \le t_{\alpha/2,n-1}\right\} = 1-\alpha\,. \tag{5}$$

Substituting for t in Equation (4) and solving for the mean μ, we arrive at a $(1-\alpha)100\%$ confidence interval for the mean

$$[\underline{\mu}, \overline{\mu}] = [\hat{\mu} - t_{\alpha/2,n-1}\, s/\sqrt{n}\,, \hat{\mu} + t_{\alpha/2,n-1}\, s/\sqrt{n}]\,. \tag{6}$$

2.2.2 Confidence interval for the variance

Since x_1, x_2, …, x_n is a random sample from a normal distribution with unknown mean μ and unknown variance σ^2, it can be shown that the sampling distribution of

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} \tag{7}$$

is chi-square with $n-1$ degrees of freedom, where n is the sample size and s^2 is the sample variance (Hines, Montgomery et al. 2003). To develop the confidence interval, we note that

$$P\left\{\chi^2_{1-\alpha/2,n-1} \le \chi^2 \le \chi^2_{\alpha/2,n-1}\right\} = 1-\alpha\,. \tag{8}$$

Substituting for χ^2 in Equation (7) and solving for the variance σ^2, we arrive at a $(1-\alpha)100\%$ confidence interval for the variance

$$[\underline{\sigma}^2, \overline{\sigma}^2] = \left[\frac{(n-1)s^2}{\chi^2_{\alpha/2,n-1}}, \frac{(n-1)s^2}{\chi^2_{1-\alpha/2,n-1}} \right]. \tag{9}$$

A table of t and χ^2 values is found in most probability and statistic books, such as (Hines, Montgomery et al. 2003).

2.3 The payoff of a decision

There are two layers to simulation-based design: deciding which models to use to guide the design decision and the design decision itself. The value of both decisions is measured by the success of the design (i.e. the design decision). The outcome of a decision problem can be represented by a *payoff function*, $\pi(x,a)$, that depends on both the chosen action a and the realized state of the world x. Because of uncertainty in the state of the world x, the DM cannot know the outcomes, or payoff, of any action with certainty.

We measure the payoff in terms of utility. As originally proposed by von Neumann and Morgenstern (von Neumann and Morgenstern 1980), utility analysis is used for making decisions under uncertainty in traditional statistical decision theory (Pratt, Raiffa et al. 1995). In general, *utility* expresses preference—more preferred decision outcomes are assigned higher utility values. If chosen correctly, utility reflects the DM's preferences even under uncertainty. By applying the expected value operator, the DM weights all possible outcomes according to their likelihood of occurring, and then chooses the action that maximizes the expected utility. For a general overview of utility theory see (Fishburn 1982; Keeney and Raiffa 1993).

2.4 Decision policies under imprecision

When imprecise probabilities are mapped through a utility function the result is imprecise utilities, i.e. intervals of utility, see Figure 4. A special class of decision policies is needed to make decisions based on imprecise utilities. The simplest of these decision policies is interval dominance, which states that any interval that is completely dominated by another interval can be eliminated from consideration. From Figure 4, we see that interval of utility for design alternative 3 is dominated by the interval of utility for both of the other design alternatives; therefore, design alternative 3 can be eliminated from consideration.

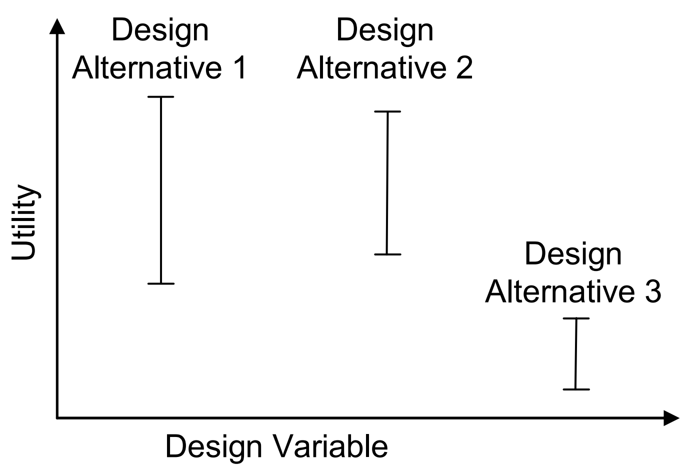

Figure 4: Imprecise utility intervals for three design alternatives

But what about making a decision between design alternative 1 and design alternative 2? Based on decision policies developed under the assumption of precision, overlapping utility intervals leads to indeterminacy. Since a decision must be made even if the utility intervals overlap, a decision policy that can resolve this ambiguity is required. Possible policies for making decisions under imprecision include maximality (Walley 1991), maximax (Berger 1985; Schervish, Seidenfeld et al. 2003), maximin (Wald 1950), E-admissibility (Schervish, Seidenfeld et al. 2003), and the Hurwicz criterion (Arrow and Hurwicz 1972; French 1988).

For this thesis we chose to use the Hurwicz criterion, which generalizes the maximax and maximin decision policies and provides a flexible framework for decisions under imprecision. The Hurwicz decision criterion uses an optimism-pessimism index, $0 \le \alpha \le 1$, which represents the DM's level of pessimism. If $\alpha = 1$, the DM is entirely pessimistic and chooses to use a maximin decision policy, while if $\alpha = 0$, the DM is entirely optimistic and chooses a maximax decision policy. Other values of α move the decision point, U^{α}, between these two extremes according to the equation $U^{\alpha} = \alpha \underline{U} + (1-\alpha)\overline{U}$ as illustrated in Figure 5.

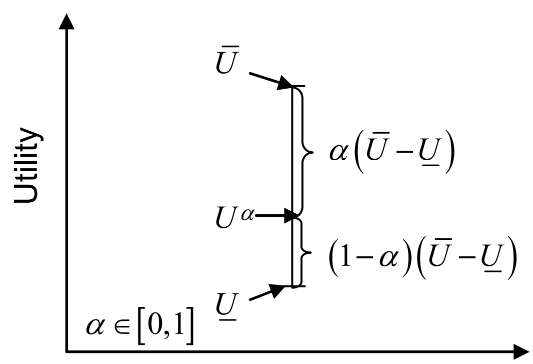

Figure 5: The Hurwicz decision criterion with decision point U^{α}

2.5 Information economics

The area of information economics grew out of statistical decision theory in the 1950s when Marschak published a series of papers on the economics of information and organization (Marschak 1974). Recently, with the explosion of new information technologies, information economics has regained attention within the broader context of information management. Current areas of research focus on corporate finance and industry policy, such as intellectual property rights, industry regulation, and fostering innovation (Rubin 1983; Strassmann 1999), or on the infusion of information technology into a corporation (Strassmann 2004). Within engineering, the focus of information management has been primarily on data exchange, interoperability, and visualization to support collaborative design. For an overview of these areas, refer to the following review articles (Ciocoiu, Gruninger et al. 2001; Jayaram, Vance et al. 2001; Rangan and Chadha 2001; Szykman, Sriram et al. 2001; Urban, Dietrich et al. 2001).

In a more general sense, information economics presents principles by which the cost-benefit tradeoffs of information collection can be managed in engineering design. The principles can be summarized by the following statement: the DM should only purchase information that has positive net value. These principles have been developed and employed previously in standard micro-economics and the theory of the economic value of information, pioneered by Marschak (Marschak 1974) and summarized by Lawrence (Lawrence 1999). A substantial difference between engineering design applications and those of Marschak and Lawrence is the availability of perfect knowledge—knowledge that Marschak and Lawrence assume to be available, but engineers often lack in practice.

2.6 Cost-benefit tradeoffs of information

As a designer collects more information, the marginal benefit of acquiring additional information decreases. For example, say the designer wishes to characterize the stress-strain curve of a novel material by testing the failure strength of material samples. If the designer has only tested 10 samples, an 11^{th} test will usually be quite valuable; in contrast, if the designer has tested 1000 samples, the 1001^{st} test will be considerably less valuable. In this sense, information displays diminishing returns. At some point, the cost of gathering additional information will outweigh the benefit. Thus, the value of a sample is not merely inherent in the sample; rather, the value is measured as viewed from the perspective of the DM. A fundamental principle of information economics is that a DM should continue to collect information only as long as there is an information source available whose net value is positive. Putting the example problem into more standard micro-economic terms, a rational DM stops collecting data samples at the point where the marginal benefit of the next sample is less than or equal to the marginal cost of acquiring it.

A formalization of the basic cost-benefit analysis noted above has been summarized in the context of information by Lawrence (Lawrence 1999). In his work, the measure by which information can be managed is *value*.

2.7 Related research in engineering design

In related research, Gupta et al. have demonstrated the importance of incorporating the cost (in terms of number of design alternatives considered) of decision making into the overall design decision model (Gupta and Xu 2002), but they do not provide an approach for estimating the value of information in actual design problems. Radhakrishnan and McAdams consider the cost-benefit trade-offs in selecting models of various levels of abstraction in engineering design (Radhakrishnan and McAdams 2005). They present a framework in which a designer can reason about model uncertainty, but they admit that the designer is left with little guidance in estimating the actual value of information from different models. Along similar lines, Bradley and Agogino develop a decision-analytic approach to assist designers in cost-benefit analysis of resource expenditures using precisely characterized probability distributions to guide and prioritize information collection (Bradley and Agogino 1994), but they do not explain how to estimate these distributions.

Howard develops a theory of the value of information which takes into account both probabilistic and economic factors in decisions and uses this theory to determine the optimal number of tests to perform to characterize a known distribution with unknown parameters (Howard August 1966; Howard November 1965). Matheson extends Howard's theory and uses it to determine the most economic computations and analyses to perform for a particular decision problem (Matheson September 1968). Although Howard's and Matheson's works are similar in objective to this thesis, their approach depends on the designer's ability to accurately assign precise probabilities to the possible states of nature before performing the analysis (i.e. having accurate priors).

In the simulation literature, statistical output analysis is commonly performed to assess whether a sufficient number of simulation replicates have been performed to obtain statistically significant conclusions (Law and Kelton 2000). However, since the analysis is performed based on accuracy requirements, one cannot easily formulate this trade-off with respect to the simulation cost. As in any kind of cost-benefit analysis (Layard and Glaister 1994), a common unit of measure is needed. This need can be met by using the *economic value of information* (Lawrence 1999).

Although the economic value of information is clearly correlated with accuracy, they are not equivalent. For example, when distinguishing between two alternatives that differ significantly in performance, a very accurate and expensive model is less valuable than a simpler model that could have made the same distinction at a much lower cost. Conversely, in high-risk design problems, an expensive model that is more accurate than typically required may lead to a better solution even when factoring in costs, since a simple model may lead to a decision with disastrous consequences.

2.8 Identification of knowledge gap addressed in this thesis

Two research communities have presented approaches for managing the cost-benefit trade-offs of information collection. The design community has developed frameworks for managing this trade-off based on strong assumptions about the amount of knowledge, either by assuming that the value of information is known, that knowledge about the outcome of information collection is known, or that the DM can specify accurate probabilities distributions prior to the collection of information. Consequently, these frameworks are not easily applied to engineering problems in which such strong assumption are invalid. The statistics community has provided an approach that uses statistical significance levels as the metric for managing the information cost-benefit trade-off, but the cost of information collection is not taken into account. We postulate that the DM is not directly interested in the increase in statistical significance that information provides; instead, he or she is interested in how much the information increases the net value of the design; net value is the benefit that the information provides minus the cost of collecting that information.

This thesis proposes approaches that use the economic value of information to manage the information cost-benefit trade-off and overcomes many of the difficulties encountered by the design community by representing the DM's knowledge using imprecise probabilities. The approaches proposed unite the information economic framework developed by Lawrence (Lawrence 1999) with the axiomatic theory for imprecise probabilities developed by Walley (Walley 1991). The P-box formalism developed by Ferson (Ferson and Donald 1998) is used to characterize imprecise probabilities in a representational and computationally tractable form. Representing the imprecision in the DM's knowledge allows us to bound the value of future information; bounds that can be used to guide the information cost-benefit trade-off. Such guidance allows for the explicit management of the information cost-benefit trade-off in a broader class of engineering design problems.

3 Managing the collection of statistical information under uncertainty using information economics*

An important element of successful engineering design is the effective management of resources to support design decisions. Design decisions can be thought of as having two phases—a formulation phase and a solution phase. As part of the formulation phase, engineers must decide what information to collect and use to support the design decision. Since information comes at a cost, a cost-benefit trade-off must be made. Previous work has considered such trade-offs in cases in which all relevant probability distributions were precisely known. However, engineers frequently must characterize these distributions by gathering sample data during the information collection phase of the decision process. This characterization is crucial in high-risk design problems where uncommon events with severe consequences play a significant role in decisions. In this chapter, we introduce the principles of information economics to guide decisions on information collection. We investigate how designers can bound the value of information in the case of distributions with unknown parameters by using imprecise probabilities to characterize the current state of information. We explore the basic performance, subtleties, and limitations of the approach in the context of characterizing the strength of a novel material for the design of a pressure vessel.

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3.1 Example Problem

Throughout this chapter, we discuss the application of information economics in the context of an example of a pressure vessel design. This example has been used previously to demonstrate the value of using imprecise probabilities in engineering design decisions (Aughenbaugh and Paredis 2005). We now extend this experiment to explore the decision of how much information to collect in order to support design decisions.

In the example problem, a pressure vessel is designed to meet certain requirements while maximizing payoff. The complication is that the pressure vessel is to be built using a material with unknown yield strength. It is assumed that the yield strength is well modeled as a normally distributed random variable, but that the parameters of the normal distribution are unknown. Yield strength tests can be performed, thus sampling the distribution at a cost c per test.

In this experiment, each yield strength test represents one sample from the true material strength distribution, a normal distribution whose parameters are unknown to the designers. Specifically, the material strength is a random variable X such that:

$$X \sim N(\mu, \sigma^2)\,. \tag{10}$$

The mean μ and variance σ^2 are unknown, and the goal of the information collection is to accurately estimate these parameters such that a good design decision can be made. The experiment consists of drawing a set of n samples $\{x_i\}_{i=1}^{n}$ from X. Each sample x_i that is drawn from the distribution is a piece of information that can be used to help characterize the true nature of the uncertainty. Unless the designers have infinite resources, they cannot collect the infinite number of samples necessary for a perfect characterization of the distribution. Instead, they need to determine when to stop collecting information—in this case, data samples.

3.2 Mathematical Problem Formulation

In engineering design, the value of information can be measured by observing how the information affects the design decision. In this section, we explain the basic principles of information economics and illustrate this framework with a simple example in which precise probability distributions are assumed.

3.2.1 Specifying probabilities over the state space

The set of all possible states of the world form a *state space* $X = \{x\}$. In the example problem, the state of the world is the actual material strength x of the material used in a particular pressure vessel. The material strength, or state, is assumed to be normally distributed with associated probability density function $p(x)$, with parameters that are unknown to the designer. The state of the world is outside the DM's control, so the DM can at best estimate the probabilities, thus forming the estimated distribution $\tilde{p}(x)$.

3.2.2 The payoff of a decision

For every decision problem, a DM has a set of available *actions* $A = \{a\}$ from which to choose one. Once an action has been chosen, the DM will receive a *payoff*, $\pi(x,a)$, that depends on the action a chosen and the realized state of the world x. In the example problem, the action a consists of a set of design variables that specify the pressure vessel dimensions. The payoff function used in the example problem, shown in Eq. (11), is highly skewed—the payoff when the vessel fails is largely negative (minus $1 million), yet the payoff when it succeeds is only slightly positive (the selling price of $200 minus the cost of the material used to build the pressure vessel). Skewed payoff functions are common in applications involving risk where uncommon events with severe consequences play a significant role in decisions. Note that for a given yield strength and design, the failure cost is either zero (no failure) or a constant (the cost of the damage, lost productivity, etc. when the pressure vessel fails).

$$\pi(a,x) = P_{selling} - C_{material} * volume(a) - C_{failure} * \delta(a,x), \tag{11}$$

where:

$P_{selling}$ ≡ selling price = \$200

$C_{material}$ ≡ material cost per volume = \$8500/m^3

x ≡ true yield strength of pressure vessel

a ≡ design variables (radius, thickness, length)

$C_{failure}$ ≡ cost incurred if vessel fails = \$1,000,000

$$\delta(a,x) \equiv \text{failure indicator} = \begin{cases} 0 & \text{if } x \geq \sigma_{\max}(a) \\ 1 & \text{otherwise} \end{cases}$$

Direct use of the payoff function in the decision implies that the DM is risk neutral. If the DM is risk-averse or risk-taking, the payoff function should be mapped to a utility function according to this risk attitude. The information economic approach presented in this chapter can be used in such situations by performing the same cost-benefit analysis in terms of utilities instead of dollars.

By choosing a precise payoff function, we have assumed perfect models of price, cost, and demand, models that do not typically exist. Imprecise value models could be used; however, this additional imprecision would translate into larger (less precise) bounds on the value of information. We chose a precise value model to limit the number of sources of imprecision to one (the material strength characterization). Limiting the sources of imprecision allows for a clearer presentation of this new approach.

3.2.3 Making an optimal decision

Because of uncertainty in the state of the world x, the DM cannot know the payoff of any action with certainty. We assume that the DM seeks to maximize the *expected payoff*, given by $E_x[\pi(a,x)]$. The expectation is taken over all states x because that is what the DM is modeling as random. Note that ideally the expectation is taken with respect to the true distribution $p(x)$. Yet, in the example (and in most real world design scenarios), the DM does not know the true distribution $p(x)$, and must instead use his or her subjective distribution $\tilde{p}(x)$. The DM thus makes an optimal decision, a^* such that

$$a^* = \arg\max_{\mathrm{a}} \ (E_{\tilde{p}(x)}[\pi(a,x)]) . \tag{12}$$

We deviate slightly from standard notation and write $E_{\tilde{p}(x)}$ to emphasize that the DM maximizes the expectation, as calculated using his or her subjective probability density function $\tilde{p}(x)$. A similar distinction must be made when determining the payoff of the decision. The true expected payoff is calculated using the true $p(x)$ that is unknown to the designer:

$$\pi_{true} = E_x[\pi(a^*,x)] . \tag{13}$$

The estimated expected payoff according to the designer's subjective distribution is

$$\pi^*_{\tilde{p}(x)} = E_{\tilde{p}(x)}[\pi(a^*,x)] . \tag{14}$$

This payoff $\pi^*_{\tilde{p}(x)}$ will in general differ from the true payoff π_{true}. Although Lawrence (Lawrence 1999) does not make this distinction in his work, the distinction is crucial in cases in which the designer has only imprecise information.

3.2.4 Information and information sources

The definition of information varies significantly by subject and application. In this chapter, we modify Lawrence's definition (Lawrence 1999) and define *information* as any stimulus that changes the recipient's subjective probability distribution $\tilde{p}(x)$ over a well-described set of states, $X = \{x\}$.

An *information source* is anything that provides information. This information arrives in the form of a *message* y taken from the probability distribution of the messages, $p(y)$. In the example problem, the information source is the yield strength testing process, and a message is the result of a single yield strength test—that is, one observation of material strength. Information economics studies whether it is valuable to pay an information source for a message. Before the message is received, a DM does not know what information that message contains, and therefore the DM does not know exactly how it will change his or her subjective probability distribution $\tilde{p}(x)$ over the state space. In turn, the DM does not know how the message will affect the decision a^* and its payoff. Thus, a DM should apply the principles of information economics to arrive at a formal metric for determining if the benefit of a message outweighs the cost of acquiring it—the *value of information*.

3.2.5 The value of information

We begin by considering two possible decisions: the first decision is made using the current state of information, and the other is made after the receipt of message y. In the first case, assume the DM's subjective probability distribution of the states is represented as $\tilde{p}(x)$. These are the *prior* probabilities, and the optimal prior decision a_0^* is given by

$$a_0^* = \arg\max_a \ (E_{\tilde{p}(x)}[\pi(a,x)]). \tag{15}$$

After the message y is received and incorporated into the DM's knowledge, the DM has an updated *posterior* probability distribution $\tilde{p}(x \mid y)$. The corresponding optimal decision a_y^* is given by

$$a_y^* = \arg\max_a \ (E_{\tilde{p}(x|y)}[\pi(a,x)]). \tag{16}$$

How can we compare these two decisions? If we wait until the true state of the world x is revealed, we can calculate the *ex-post gross value of the message* y—where *gross* implies *before* factoring in cost—for the particular realized state x as:

$$\upsilon\left(y \mid x\right) = \pi(a_y^*, x) - \pi(a_0^*, x). \tag{17}$$

This represents the amount that the receipt of message y (and the incorporation of its information into the decision) changed the DM's payoff, given the particular outcome x of the state.

The term *value* is used throughout this chapter in a *marginal* sense, that is, in terms of differences. The ex-post gross value of a message y is the *marginal payoff* of acquiring that message—the difference between the payoff of the decision with and without the information from message y. This gross value can be positive, negative, or zero. It is positive if the message leads the DM to chose an action a_y^* that has a higher payoff under realized state x than action a_0^*. It is negative if the message in someway misled the DM into choosing an action a_y^* that has a lower payoff than the prior decision a_0^*. If the message did not change the choice of action, such that a_y^* is the same as a_0^*, then the ex-post gross value is zero.

The previously defined ex-post gross value is not useful for determining the potential change in payoff of receiving a message because it measures the actual benefit, which can only be known *after* the decision is made and the truth realized. It is common knowledge that a good decision can lead to a bad outcome, especially if a rare, adverse state of the world is realized—a situation referred to in the vernacular as *bad luck*. Conversely, a bad decision can lead to a good outcome—a case of *good luck*.

Rather than assessing the value of a message for a particular state x, a DM is really interested in the expected value over all the possible states of the world. The *gross value* of a message y is defined as the expected difference in the payoff with and without the message, such that:

$$\upsilon(y) = \text{gross value}(y) = E_x[\pi(a_y^*, x) - \pi(a_0^*, x)]. \tag{18}$$

Calculating the true gross value of a message requires the expectation over the true distribution $p(x)$, which is not available to the DM.

To complicate matters further, Eq. (18) is valid for analysis of the value of a particular message y only after it is received. However, when the DM needs to decide whether or not to purchase a message, the content of the message—that is the particular message y from the set $\mathbf{Y}$ of all possible messages distributed according to some $p(y)$—is also unknown. When purchasing a message, it is as if the DM is purchasing a sealed envelope; he or she does not know what is inside until after buying and opening the envelope. The DM must therefore consider the value of the information source $\mathbf{I}$ instead of the value of a single message.

If the DM had access to the true probability distribution of the messages, $p(y)$, over the set $\mathbf{Y}$, he or she could calculate the gross value of the next message from an information source $\mathbf{I}$:

$$\text{gross value}(\mathbf{I}) = E_y E_x[\pi(a_y^*, x) - \pi(a_0^*, x)]. \qquad (19)$$

Because the DM does not have access to the parameters describing the true probability distribution of the messages $p(y)$ or of the states $p(x)$, Eq. (19) cannot be used directly to estimate the value of an information source. In this chapter, we investigate an approach for bounding the value of information that incorporates the imprecision of the DM's information state.

A final definition that ties our notion of value back to the fundamental concept of cost-benefit analysis in information economics is *net value*. A message y must be purchased at some cost; resources need to be expended in order to acquire more information. Denoting this as $\text{cost}(y)$, the *net value* of a message is defined as

$$\text{net value}\left(y\right) = E_x[\pi(a_y^*, x) - \pi(a_0^*, x)] - \text{cost}\left(y\right). \tag{20}$$

Similarly the net value of the next message from an information source is:

$$\text{net value}(\mathbf{I}) = E_y E_x[\pi(a_y^*, x) - \pi(a_0^*, x)] - \text{cost}(\mathbf{I}), \tag{21}$$

where $\text{cost}(\mathbf{I})$ is the cost of receiving one message y from information source $\mathbf{I}$.

If we revisit the DM's goal of making a cost-benefit tradeoff during information collection, we can now state the information economic principle that a designer should purchase a message from an information source if the net value of that information is positive. According to Eq. (21), this requires the calculation of expectations across the distributions $p(x)$ and $p(y)$, which in general are not known to a designer. We return to the problem of not knowing $p(x)$ and $p(y)$ after illustrating the simpler case of known probabilities.

3.2.6 Example with known probabilities

In this section, we present an example to illustrate the calculation of value of information in the hypothetical case of known probabilities. We later extend this example to the more practical case of unknown probability parameters. While the information used in this example is not available to a DM, it is useful for illustrating the basic approach, shown in Figure 6.

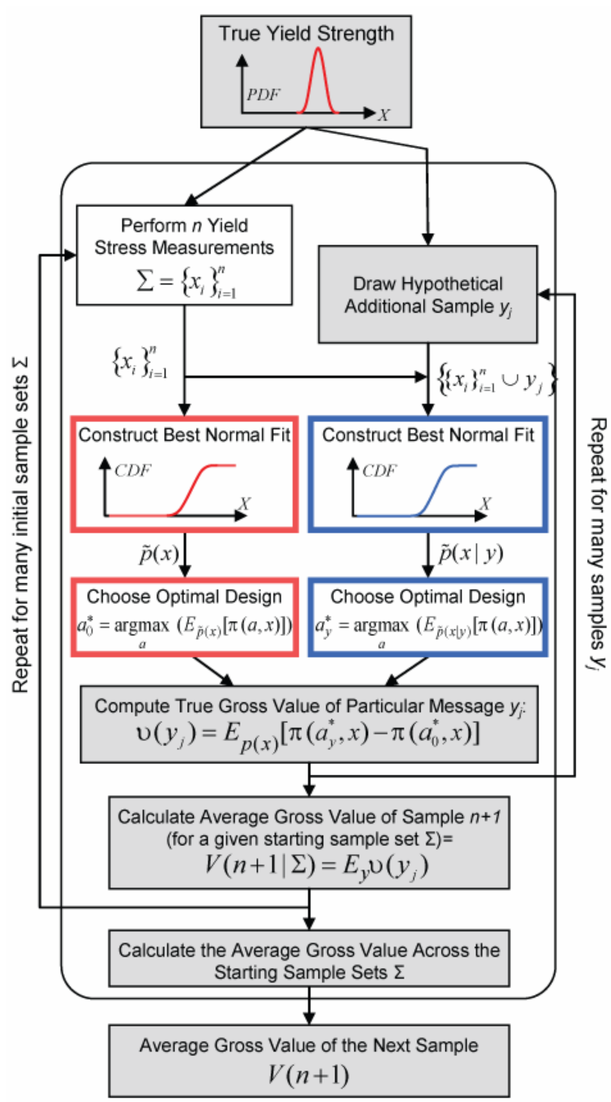

Figure 6: Overview of approach using known probabilities to calculate the value of information

We assume that there is an omniscient supervisor overseeing the experiment. This supervisor knows the true distribution and can perform the actions shown in the gray boxes. These actions are normally not available to the DM. In this approach, the DM begins with the observed set of samples $\Sigma = \{x_i\}_{i=1}^{n}$. The goal is to determine whether it is valuable to collect an $(n+1)^{st}$ sample given the existing n samples. The DM first uses this set of samples to construct a best-fit distribution $\tilde{p}(x)$, and then to choose an optimal design a_0^*, as shown on the left side of the figure. The DM then receives a hypothetical additional sample y_j from the supervisor. The DM constructs a new best-fit distribution $\tilde{p}(x \mid y_j)$ and makes a new decision $a_{y_j}^*$. The difference in expected payoffs of the two decisions is then calculated by the supervisor to determine the true expected gross value $\upsilon(y_j)$ of the particular message y_j. This process of calculating the value of an additional sample is repeated over many y_j to calculate the average value of the next sample for a particular starting set of n samples, which we denote as $V(n+1 \mid \Sigma)$.

Recall that the net value of the next piece of information depends on the prior decision a_0^*, which in turn depends on the existing data samples. For example, the net value of purchasing an 11$^{\text{th}}$ sample from the information source depends on the first 10 samples. If the initial 10 samples just happen—by chance—to yield very good estimates of the distribution parameters, then the net value of the 11$^{\text{th}}$ sample will be small, but if they yield bad estimates of the distribution parameters, then the net value of the 11$^{\text{th}}$ sample could be large. Consequently, the next step is to repeat the experiment over many initial sample sets Σ, which gives the average gross value of the next sample, denoted $V(n+1)$.

The final step of the experiment is to repeat the process for different initial sample *sizes*. By repeating the calculation over many initial sample sizes, we can construct a curve of the average net value of an additional sample at different sample sizes, as shown in Figure 7. This figure can be interpreted as follows: at a prior sample size n=32, the average net value of an additional sample (the 33rd sample in this case), is about $2. The net value of the 52nd sample, starting from 51 samples, is negative, but the net value of the 51st sample is positive. This means that the 52nd sample is the first sample whose average net value is negative; therefore, by stopping at 51 samples the designer will achieve the highest expected utility. Note that this conclusion is drawn using the true $p(y)$ and $p(x)$, which are not available to the DM.

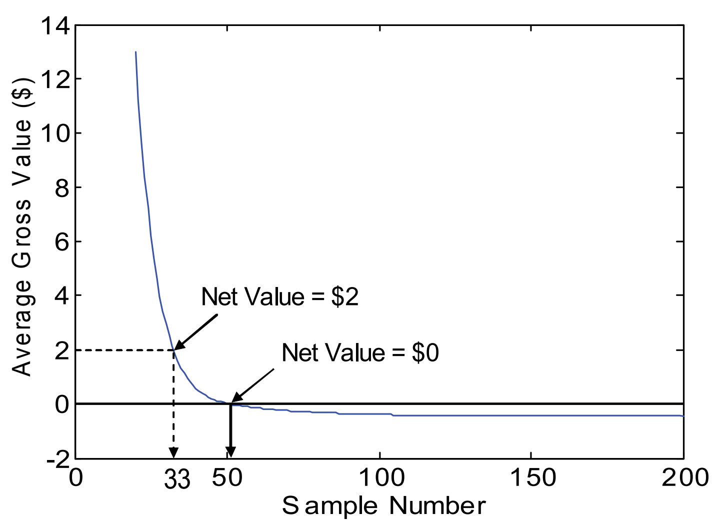

Figure 7: Net gain in payoff per sample

The results can also be interpreted by considering the net expected payoff, which is the payoff of the design that would have been realized if no additional information were collected, less the cost of the already collected n samples:

$$\text{net expected payoff} = E_{p(x)}[\pi(a^*, x)] - n \cdot \text{cost}(y) \tag{22}$$

The results are shown for different sample sizes in Figure 8. Again, because the actual observed samples affect the payoff, the payoff of the design is averaged over many initial sample sets. The relationship between this result and the net value of additional samples should be clear; the maximum net expected payoff occurs at the same sample size at which the net value of an additional sample first becomes negative. Recalling that the net value is defined in a marginal sense, moving from 51 samples to 52 samples means a decrease in total payoff of the decision, as is revealed in both plots.

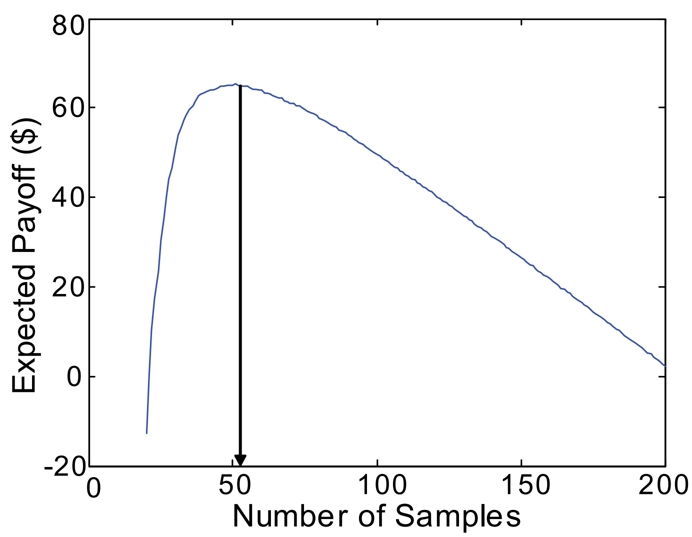

Figure 8: Net expected payoff of the design

In the preceding analysis, it appears simple to determine the optimal number of samples to collect. However, this simplicity is due to the omniscient supervisor having precise knowledge of the true distributions $p(y)$ and $p(x)$. In the example problem the information source is an unbiased model of the truth, which means that $p(y)$ and $p(x)$ are identical yet unknown—they both describe the unknown true material strength. The characterization of $p(x)$ is the DM's indirect goal for data collection—the DM wants to characterize $p(x)$ well enough that the design based on the estimated $\tilde{p}(x)$ is acceptable.

To determine the value of information during the actual design process, the DM needs an approach by which he or she can estimate the net value of an additional data sample when the parameters describing $p(y)$ and $p(x)$ are unknown. We propose an approach that uses imprecise probabilities to calculate an interval of net value for an additional sample. What performance characteristics should we expect or demand of this approach? Insight can be gained by examining the distribution of the net payoffs about the expected value curve of Figure 8. Box plots for sample sizes of 50, 100, and 150 are shown in Figure 9. The plots are constructed with the whiskers at the 0.1% and 99.9% quantiles, and the boxes from 25% to 75%. The extreme skewness of the box-plots is due to the skewed payoff function; that is, the cost of slightly under designing the pressure vessel is large compared to the cost of slightly over designing it. The box plots reveal that both the variance of the payoff and the chance of a catastrophic result decrease as the sample size increases but that, simultaneously, the expected net value decreases significantly. The behavior shown in Figure 9 suggests that a reasonable estimation of the optimal number of samples (when the DM has only imprecise knowledge about the true distribution) is often well beyond 51 (the optimal stopping point based on expected value), because by stopping at 51 samples, a DM still faces a very large downside risk. It is important to consider the distributions in Figure 8 and Figure 9 when developing an approach for determining the value of additional samples; however, in practice, an engineer does not have this information available for decision making. We return to this issue after introducing how we will use imprecise probabilities.

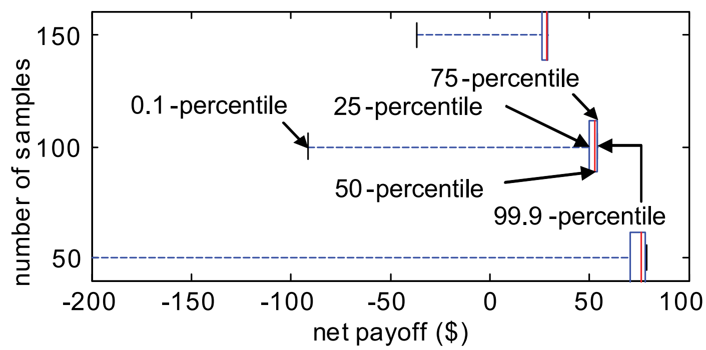

Figure 9: Box plots for various sample sizes

3.3 Imprecise probabilities

In this chapter, we use a probability-box or *p-box* (Ferson and Donald 1998), to represent imprecise cumulative probability distributions as was explained in the imprecise probabilities section of the Background chapter. While there are several methods to construct p-boxes (Ferson 2002), we choose a practical method based on traditional confidence intervals on the mean and variance (Aughenbaugh, Ling et al. 2005):

$$[\underline{\mu}, \overline{\mu}] = [\hat{\mu} - t_{\alpha/2,n-1}\frac{s}{\sqrt{n}}, \hat{\mu} + t_{\alpha/2,n-1}\frac{s}{\sqrt{n}}] \tag{23}$$

$$\left[\underline{\sigma}^2, \overline{\sigma}^2\right] = \left[\frac{(n-1)s^2}{\chi^2_{\alpha/2,n-1}}, \frac{(n-1)s^2}{\chi^2_{1-\alpha/2,n-1}}\right] \tag{24}$$

By choosing a particular confidence level for the mean and variance intervals, a DM is essentially stating that he or she is comfortable assuming that the true distribution lies entirely in the resultant p-box. This assumption is similar to accepting the p-box as a model of the truth. This distinction becomes important in our approach for estimating the value of information, as explained in the following section.

3.4 Estimating the value of information

In this section, we explain our approach to bounding the gross value of the next message from an information source. We start by describing how design decisions are made. We then motivate the use of imprecise probabilities, describe our approach for estimating the value of information, and present a computational experiment that illustrates the results of our approach.

3.4.1 Design decision policy

According to Eq. (12), the DM chooses the design action that maximizes the expected payoff, with the expectation calculated using $\tilde{p}(x)$. This distribution is derived by assuming that the material strength is normally distributed and then using the sample mean and sample variance of the observed samples as precise estimates of the true mean and variance. Other work has presented a decision policy that incorporates imprecision into $\tilde{p}\left(x\right)$ during the solution phase of the design decision (Aughenbaugh and Paredis 2005), much as the approach in this chapter incorporates imprecision into the problem formulation phase. Nevertheless, for this chapter a decision policy based on a best-fit distribution is used in the problem solution phase in order to isolate the effect of accounting for imprecision in the problem formulation phase—that is, to emphasize the contributions of applying information economics. A noted item for future work is the combination of these approaches into one unified approach that explicitly considers imprecision throughout the design process.

3.4.2 Motivation for imprecise probabilities

One motivation for using imprecise probabilities to represent the DM's state of information is that the use of precise probabilities does not enable useful estimates of value. The necessity of an alternative to precise probabilities is illustrated in the following example. Assume that the DM represents his or her state of information $\tilde{p}(x)$ precisely. Using this information, the DM chooses an optimal design a_0^* according to Eq. (15), using $\tilde{p}(x)$ when evaluating the expectation.

Now assume that the DM acquired an additional data sample y. With this information, the DM can create a new subjective distribution $\tilde{p}(x \mid y)$, where in general $\tilde{p}(x \mid y) \neq \tilde{p}(x)$. The DM would then choose an optimal design a_y^* according to Eq. (16), using $\tilde{p}(x \mid y)$ when evaluating the expectation.

If the DM wanted to calculate the gross value of this message y, he or she would use Eq. (18), repeated here for clarity:

$$\upsilon(y) = \text{gross value}(y) = E_x[\pi(a_y^*, x) - \pi(a_0^*, x)]\,. \tag{18}$$

Ideally the expectation E_x would be taken over the true $p(x)$, but the parameters of this distribution are unknown. The DM's two best options for approximating $p(x)$ are $\tilde{p}(x)$ and $\tilde{p}(x \mid y)$.

If the DM uses $\tilde{p}(x)$ as the best estimate of $p(x)$, we can adopt our notation from earlier and rewrite Eq. (18) as:

$$\upsilon(y) = E_{\tilde{p}(x)}[\pi(a_y^*, x) - \pi(a_0^*, x)] \tag{25}$$

or, by distributing the expectation as:

$$\upsilon(y) = E_{\tilde{p}(x)}[\pi(a_y^*, x)] - E_{\tilde{p}(x)}[\pi(a_0^*, x)]. \tag{26}$$

According to Eq. (15), the design decision a_0^* maximizes $E_{\tilde{p}(x)}[\pi(a, x)]$, thus

$$E_{\tilde{p}(x)}[\pi(a_y^*, x)] \le E_{\tilde{p}(x)}[\pi(a_0^*, x)]. \tag{27}$$

This means that the gross value of message y is always estimated to be zero or negative, no matter how much new information is available. Yet intuitively, the gross value of additional information should often be positive—acquiring information should improve the DM's ability to make a good decision on average.

If the DM instead used the posterior distribution $\tilde{p}(x \mid y)$, we can rewrite Eq. (18) as:

$$\upsilon(y) = E_{\tilde{p}(x|y)}[\pi(a_y^*, x) - \pi(a_0^*, x)]. \tag{28}$$

Expanding the expectation, we find

$$\upsilon(y) = E_{\tilde{p}(x|y)}[\pi(a_y^*, x)] - E_{\tilde{p}(x|y)}[\pi(a_0^*, x)]. \tag{29}$$

According to Eq. (16), design decision a_y^* maximizes $E_{\tilde{p}(x|y)}[\pi(a, x)]$, thus we find:

$$E_{\tilde{p}(x|y)}[\pi(a_y^*, x)] \geq E_{\tilde{p}(x|y)}[\pi(a_0^*, x)]. \qquad (30)$$

In this case, the gross value is always calculated to be positive or zero, which is also unreasonable; there will always be "unlucky" samples, or messages, that lead to a worse design. Another objection to using the precise $\tilde{p}(x \mid y)$ is that it has no use in decision making, because $\tilde{p}(x \mid y)$ is only available after the information message y is collected.

This exercise illustrates that an information collection policy based upon the assumption of precisely characterized knowledge about the true distributions is not useful. The principles of information economics cannot be applied meaningfully while using precise probabilities, but they can be implemented using an approach based on imprecise probabilities that provides useful bounds on the value of information, as described in the next section.

3.4.3 Bounding the value of information

An overview of our approach is shown in Figure 10. The DM begins with the actually observed set of data samples $\Sigma = \{x_i\}_{i=1}^{n}$. The DM first uses this sample to construct a best-fit normal distribution and to choose an optimal design a_0^*—the left side of the figure. The DM then uses the observed samples to construct a p-box using Eq. (23) and Eq. (24).

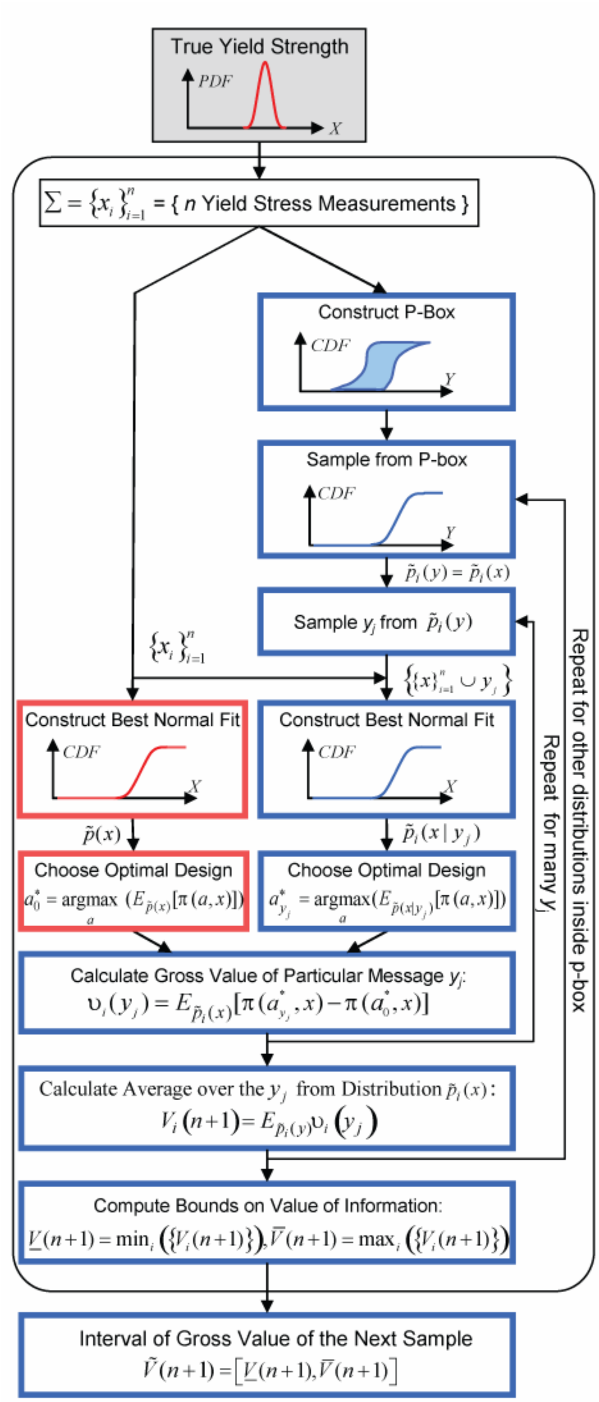

Figure 10: Overview of approach using imprecise probabilities to bound the value of information

Recall that by assumption, this model of information—the p-box—contains the CDF of the true distribution $p(x)$. The DM discretizes the p-box, as described below, and selects a single normal distribution from the p-box to represent both $\tilde{p}_i(x)$ and $\tilde{p}_i(y)$. Because the information source is an unbiased model of the truth in the example problem, these two distributions are identical; they both describe the unknown true material strength. This selected distribution is used to estimate the gross value of collecting an additional piece of information through the use of Eq. (19) with $p(x)=\tilde{p}_i(x)$ and $p(y)=\tilde{p}_i(y)$. If the DM repeated this for *every* normal distribution inside the p-box, one of the calculated values would be the true gross value of the next piece of information. Clearly, the DM cannot try every distribution, so we propose the following procedure.

The DM can partition the p-box into a finite, representative set of distributions. This is done by discretizing the confidence intervals on the mean and variance. The DM pairs all the combinations of mean and variance, resulting in a set of distributions such as shown in Figure 11. Future work will explore more efficient methods for this partitioning such as concepts from design of experiments, direct manipulation of the p-boxes, or random sampling across the confidence intervals; for illustration of the concept this method suffices.

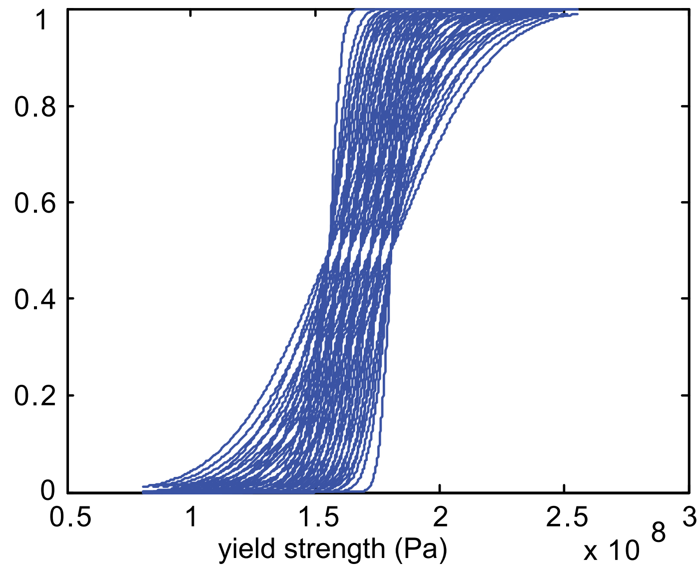

Figure 11: Various distributions in the P-box

The DM proceeds by selecting one distribution, say $\tilde{p}_i(x)$, from this finite set and assuming that this distribution is the true distribution $\left(\tilde{p}_i(x) = p(x)\right)$. The DM then calculates the gross value of taking the $(n+1)^{st}$ sample, denoted $V_i(n+1)$, via a Monte Carlo simulation, as follows.

Given the assumed message distribution, $\tilde{p}_i(y) = \tilde{p}_i(x)$, the DM can draw a hypothetical next sample, y_j, from this distribution. This sample is used, along with the actually observed samples $\{x_i\}_{i=1}^{n}$, to estimate a new posterior distribution $\tilde{p}_i(x \mid y_j)$. The DM uses this distribution to choose the optimal design, $a_{y_j}^*$, for the given distribution and hypothetical sample. The DM then evaluates the expected payoff of this design using the assumed $\tilde{p}_i(x)$, and calculates the gross value $\upsilon_i(y_j)$ of that particular y_j. The DM repeats this for many different y_j's drawn from $\tilde{p}_i(y)$ and calculates the average, or expected, gross value of the next message $V_i(n+1)$ with distribution $\tilde{p}_i(x)$ assumed to be the true distribution. Finally, the DM repeats this process for all $\tilde{p}_i(x)$ in the chosen set (from the p-box). This results in a set of gross values $\{V_i(n+1)\}$.

Recall that if the p-box had been sampled densely, then one of the values $V_i(n+1)$ in this set would be the true gross value of the $(n+1)^{st}$ sample, given the previously observed n samples. The set $\{V_i(n+1)\}$ would then form an interval $V(n+1)=[\underline{V}(n+1),\overline{V}(n+1)]$. In our approach, the p-box is only finitely sampled, so the set of values $\{V_i(n+1)\}$ only gives an approximate interval, $\tilde{V}(n+1)=[\underline{V}(n+1),\overline{V}(n+1)]$, with the lower and upper-bounds defined as $\underline{V}(n+1)=\min_i(\{V_i(n+1)\})$ and $\overline{V}(n+1)=\max_i(\{V_i(n+1)\})$. The accuracy of this estimated interval improves as the density of sampling from the p-box increases.

Based on this interval of value for the next sample, the DM decides if another sample should be taken. If another sample is taken, the process repeats itself starting with the larger set of $n+1$ data samples $\Sigma=\{x_i\}_{i=1}^{n+1}$. It should be noted that in general the p-box and hence the discretized distributions $\tilde{p}_i(y)=\tilde{p}_i(x)$ used in the analysis will be different for this new data set.

3.5 Computational Experiment and Results

We now apply the principles of information economics to the design of a pressure vessel. The experiment proceeds according to the approach shown in Figure 10 and is repeated for sample sizes up to 200. This generates intervals on the gross value for one particular sequence of random samples $\{x_i\}$. This experiment is then repeated many times to generate multiple sample traces.

Using the approach described above, we can find the bounds on the gross value of the next piece of information, $\tilde{V}(n+1)$. A graph of these bounds for a particular sequence of samples $\{x_i\}$—a particular sample *trace*—is shown in Figure 12. A trace represents the bounds on the gross value of the n^{th} sample, given a particular set of $n-1$ previously observed samples $\{x_i\}_{i=1}^{n-1}$. Figure 13 shows the upper-bound, lower-bound, and midpoint for two additional traces in the vicinity of their crossing of the cost line—the zero net value point. The curves in the two figures reveal several interesting characteristics, as discussed in the following sections.

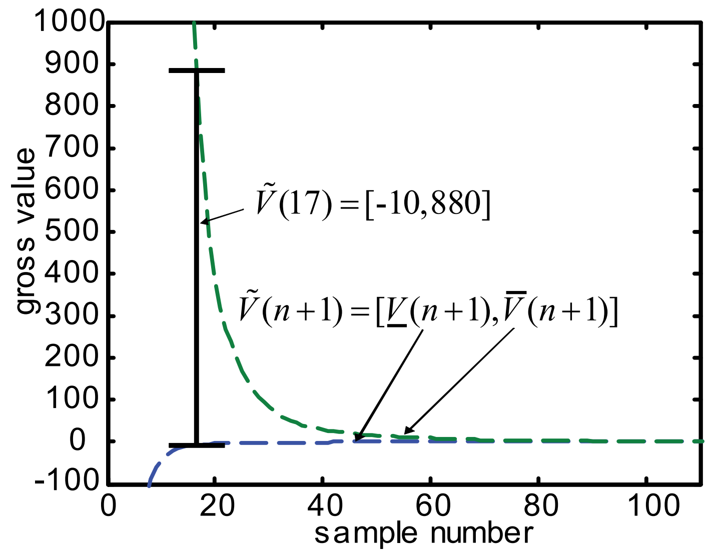

Figure 12: Example high-level behavior of gross value

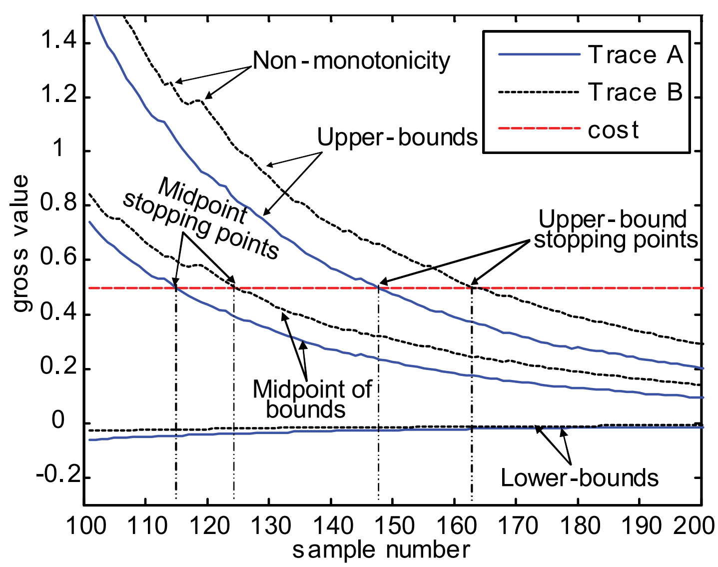

Figure 13: Two example traces of gross value

3.5.1 Small sample sizes yield large value intervals

Figure 12 shows that the potential value of the next sample for small sample sizes covers a very large range that is skewed towards the positive side. For example, the gross value of the 17th sample is in the interval [-10, 880]. Based on traditional decision policies, an interval bounding zero leads to indeterminacy. Since a decision must be made, a decision policy that can resolve this ambiguity is required. We assume one extreme—the DM stops collecting data when the upper-bound on the gross value is less than the cost—that is, when the upper-bound on the net value is negative. At the accepted confidence level, the true value is assumed to lie in the interval, so this represents the point at which the true net value cannot exceed zero, therefore, no rational DM would take an additional sample. This is a so-called *maximax* policy (Berger 1985; Schervish, Seidenfeld et al. 2003). Other possible policies for managing interval-based decisions include maximality (Walley 1991), maximin (Berger 1985), E-admissibility (Schervish, Seidenfeld et al. 2003), and the Hurwicz criterion (Arrow and Hurwicz 1972).

3.5.2 The bounds on value are not monotonic

In a general sense, it is reasonable to expect that the value of additional samples would decrease as n increases. However, each trace represents one sequence of actually observed samples. Thus, the gross value of the i^{th} sample is estimated based on the first $i-1$ samples. Once the i^{th} sample is collected, the value of the $(i+1)^{st}$ sample is calculated using all i acquired samples. If the actually acquired i^{th} sample is really "lucky" or "unlucky", the gross value of the next sample can change significantly, potentially yielding non-monotonic bounds. An example of such non-monotonicity is labeled in Figure 13. Non-monotonicity can result in multiple cost line crossings, but these crossings were never observed to be more than a few sample sizes apart. Because the bounds are already estimates, a deviation of a few samples is not significant.

3.5.3 The lower-bound is always non-positive

It is worth noting that the lower-bound on the interval will always be non-positive, i.e., given the available information, it is always possible that the gross value of the next piece of information will be less than zero. This happens because the best-fit distribution $\tilde{p}(x)$ on which the design decision is based is always contained in the set of distribution samples from the p-box—it is a candidate for the truth in our approach. This means that during the calculation of the interval on gross value, this distribution is considered as the truth at some point, yielding the situation described in Eq. (25)—if the DM's estimate already is the true distribution, which is possible though rare, then no information can make the estimate any better; it will in fact often make the estimate worse.

3.5.4 Examining the net value

The next point to note is the relationship between the gross value and cost. In practice, there is a relationship between the number of pressure vessels being designed and the cost, because the cost of information collection is amortized over all the pressure vessels. In this example, we assume that each yield strength test on a material sample costs \$0.50 per pressure vessel, and we proceed to discuss the design of one pressure vessel. Other cost functions could be used without adding significant complexity. With the cost fixed at \$0.50, an experiment following Trace A and using the upper-bound decision rule will stop with the 147th sample, because the upper-bound on the gross value of the 148th sample is less than the cost, as can be seen in Figure 13. The same logic can be applied to Trace B. In this case, a DM would collect 162 samples. At this point, the upper-bound on the gross value of the 163rd sample is less than the cost, so the net value is negative.

In this section, we have presented two representative results. The overall results consist of many sample traces that can be analyzed in the same way as the examples above.

3.6 Comparison of realized payoffs

With representative results presented in the previous section, we now move to a more general discussion. We examine the realized payoffs based on the described approach and discuss alternative decision policies for resolving ambiguity.

Using the true material strength distribution $p(x)$—which is *not* known by the designers—an omniscient supervisor could evaluate Eq. (22) to determine the actual expected payoff of the optimal design, a^*, after each sample. The results of this evaluation for Trace A from Figure 13 are shown in Figure 14. Each point represents the true expected net payoff (y-axis) of a design chosen based on the current number of samples (x-axis). Figure 14 is similar in nature to Figure 8 in the Example with known probabilities section; the volatility of the curves in Figure 14 is due to the fact that we are investigating the value along a single trace instead of averaging the value of the next sample over many traces as was shown to Figure 8. Because a DM would never create all of these designs and does not have access to $p(x)$, this is a hypothetical exercise that only the omniscient supervisor can perform using the true distribution $p(x)$.

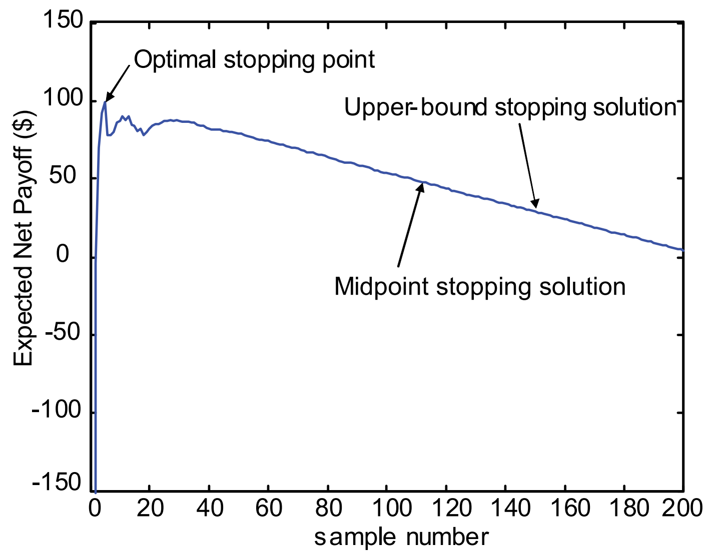

Figure 14: Actual expected net payoffs for Trace A

The results in Figure 14 indicate that, given the actual observed sequence of samples, the DM would have been best-off stopping earlier (at 5 samples) than our approach shows (at 147 samples). In this example, the DM loses about 60% of the payoff by collecting the additional 142 samples.

Is this a result of the stopping policy? Because the DM stops collecting information only when he or she is absolutely sure that the value of the next piece of information is less than its cost, the maximax decision policy is often overly conservative. An alternative policy would be to use the midpoint of the bounds, a special case of the Hurwicz criterion (Arrow and Hurwicz 1972). Using this stopping rule the DM would collect 114 samples, for Trace A in Figure 13. This still results in a loss of 50% payoff from the optimal.

Is such a loss in payoff justified? In the discussion surrounding the distribution of payoffs and Figure 9, we conclude that the DM may wish to go beyond the average "optimal" stopping point due to the imprecision in the DM's knowledge and the large downside risk of stopping too soon. The actual expected net payoffs for trace B from Figure 13 are shown in Figure 15. In this example trace, it turns out that given the actually observed samples, it would have been much worse to stop after 80 samples as compared to 100. According to Figure 13, the midpoint stopping rule would have stopped at 124 samples for this trace. While this is still about 50% below the optimal, it yields a significantly better result than a policy that would have stopped at 80 samples.

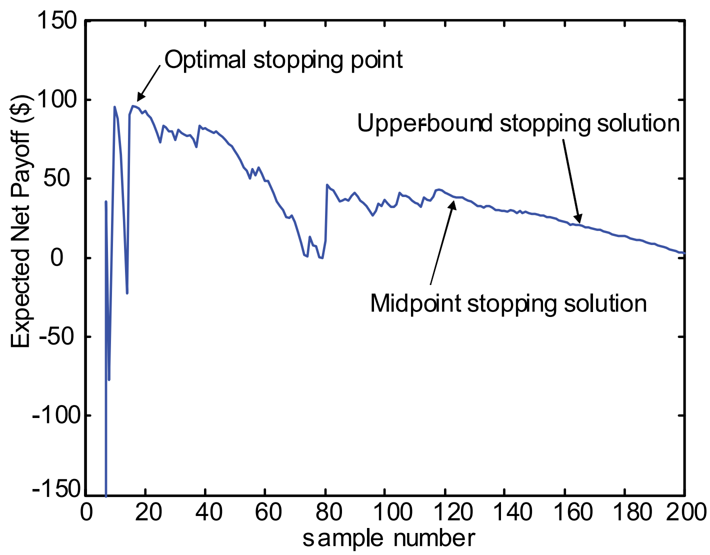

Figure 15: Actual expected net payoffs for Trace B

Before ending this analysis, we present one last trace in Figure 16**.** For this trace, the optimal stopping point would have been at 110 samples. In this case, the solution using the midpoint stopping rule of 130 samples is relatively close, though still resulting in some payoff loss. What causes the optimal stopping point to be so high in this case?

One contributing factor is that the first five actual samples were 192 MPa, 200 MPa, 194 MPa, 197 MPa, and 181 MPa. These are all above the true mean of 180 MPa. This initial "unlucky" bias leads to a severe over-estimate of the material strength, which in turn leads to a severe under-design of the pressure vessel. Consequently, the pressure vessel fails much more often than expected, leading to a significantly higher average failure cost. This example indicates how sensitive the design can be to the sample data, and why a large number of samples may be needed to reach a stable result.

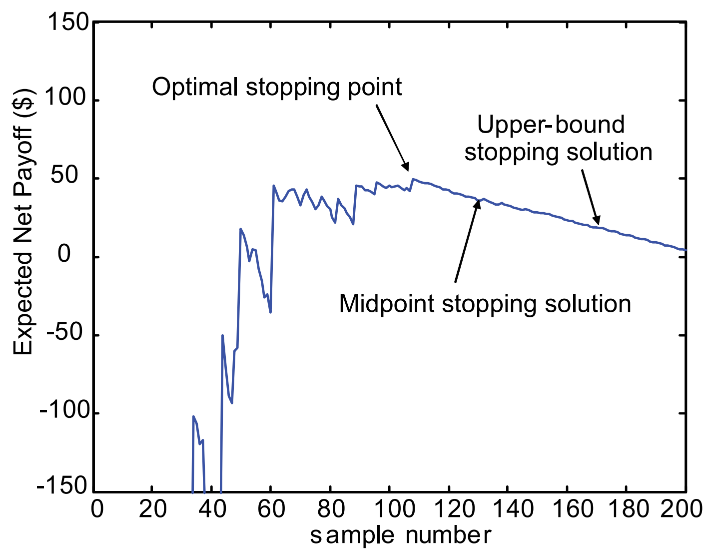

Figure 16: Actual expected net payoffs, additional trace

Before reaching a conclusion on the effectiveness of this approach of bounding the value of information, we must emphasize that the DM would not have the actual expected payoff curves available. Therefore, the DM does not know if he or she is in an example similar to that of Figure 14, Figure 15, Figure 16, or something else altogether. A conservative policy therefore leads the designer to keep taking samples until he or she is reasonably assured that there is no chance of a large negative payoff; that is, samples are taken until the downside risk is acceptable.

3.7 Summary

In this chapter, we have introduced the principles of information economics and related them to engineering design problems in which statistical parameters describing distributions are not fully known. The main contribution of this work is the investigation of an approach by which the bounds on the value of information can be calculated by a designer during the information collection process using imprecise probabilities. An open question is how to make a decision given these bounds on value. We have presented the approach and explored several example situations and decision policies. In Chapter 5, we describe the limitations of the approach and identify areas for future work.

4 An information economic approach for model selection in engineering design

In Chapter 3, the principles of information economics were used in an approach for guiding decisions about statistical data collection in the design process (Ling, Aughenbaugh et al. 2006). In this chapter, an information economic approach is used to guide *model selection* in support of engineering design decisions. This approach is illustrated with an I-beam structure design. The primary contribution of this chapter is the development of an approach for bounding the value of more accurate models using imprecise probabilities (Walley 1991).

Throughout this chapter, for simplicity, expected utility is referred to as utility. If there is no probabilistic uncertainty involved in the design, then utility and expected utility are equivalent. In such cases, the DM makes decisions based on intervals of utility. If there is probabilistic uncertainty in the design, which is often the case, then the output of the utility analysis will be intervals of expected utilities. Decisions based on intervals of utility and intervals of expected utility are made in an identical fashion, so there is no loss in generality by using the designation of utility.

4.1 Problem definition

This section provides a definition of the decision problem that is addressed in this chapter. This decision problem is an idealization of common engineering design problems; it assumes that the level of accuracy and the cost of the models are known in advance of acquiring and using the models. This section provides a definition of a model and then explains how models are used to guide decisions.

4.1.1 Modeling definitions

A model is any incomplete representation of reality (Buede 2000). In engineering design, models are typically based on theory or extensive empirical data; however, even with such a rigorous development, it should be acknowledged that "all models are wrong but some are useful" (Box 1979).

In this chapter, we assume that model output consists of bounds on the performance of the design as a function of a design parameter. These bounds are derived directly from the model output and the model error. Additionally, we assume that the model error is quantified by a multiplicative constant. A multiplicative representation of error was chosen because it is the most common in engineering models. Multiplicative error is typically represented by a statement similar to: this model is accurate to nominal value ± 5%, which indicates that the difference between the predicted and actual value is at most 5%, see Equation (31). Throughout the analysis we assume that such accuracy claims are truthful.

$$True\ Value \in \left[\left(1-\varepsilon\right) f\left(x\right), \left(1+\varepsilon\right) f\left(x\right) \right]. \tag{31}$$

where,

$f\left(x\right)$ is the output of the model

ε is the level of accuracy of the model

4.1.2 Decisions

The output of models is typically used to guide decision making. The DM uses a utility function to map model output into intervals of utility, Figure 17. Once utility intervals are known, the DM must either make the design decision based on the utility intervals, a decision under imprecision, or choose to develop a more accurate model.

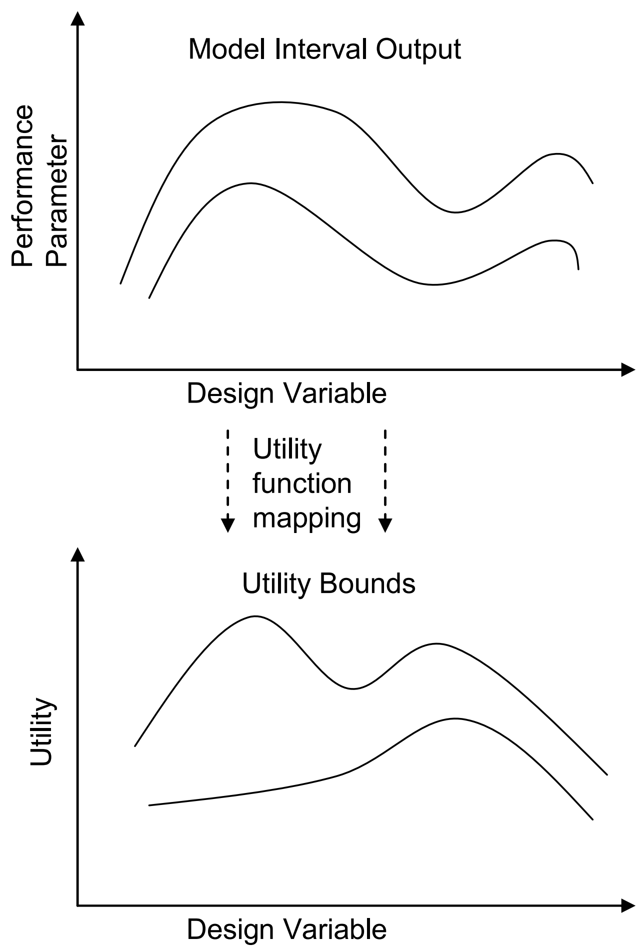

Figure 17: Imprecise model output mapped to intervals of utility

When a decision under imprecision is made, the DM must select the most desirable interval of utility by using some decision policy that can handle imprecision. Based on this decision policy and the utility bounds shown in Figure 17, suppose the DM would make the decision shown in Figure 18.

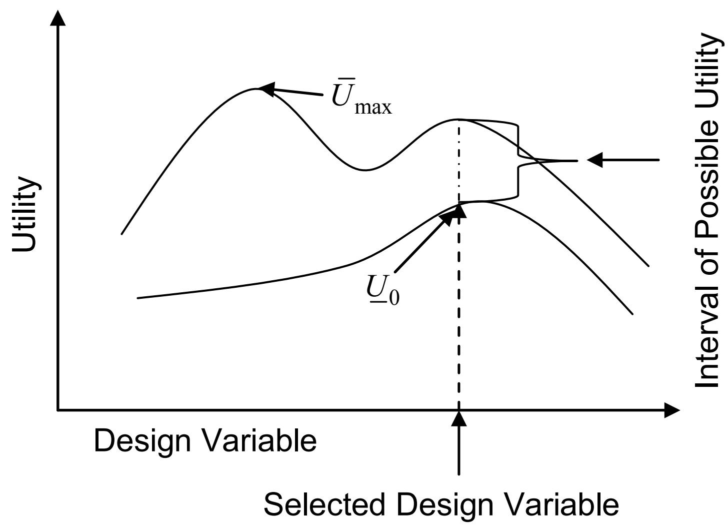

Figure 18: A decision made under imprecision

What if the decision policy led to a non-optimal decision given the precise utility curve, as shown in Figure 19? In this case, the DM did not make the decision yielding the highest expected utility and therefore could have possibly improved the decision through the use of a more accurate model. The DM paid the opportunity cost, shown in Figure 19, for making a decision under imprecision.

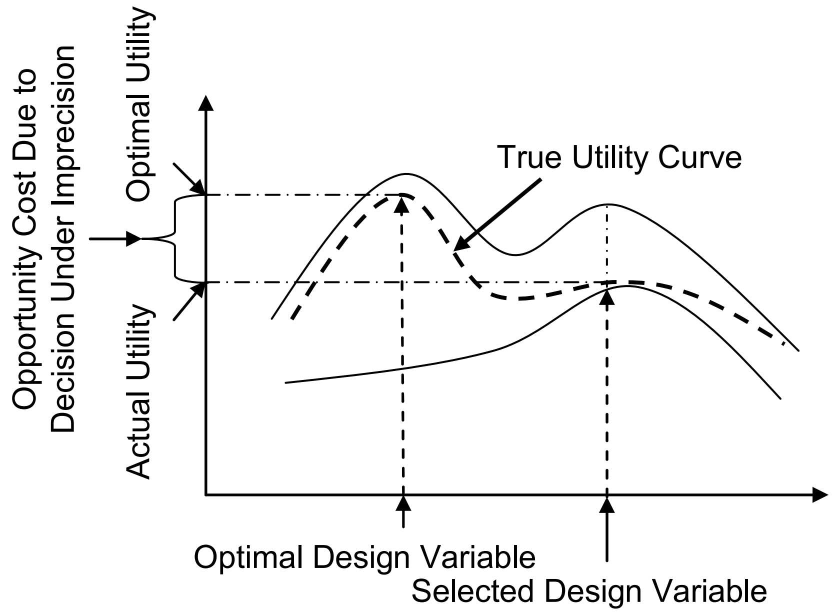

Figure 19: The loss in utility due to a decision under imprecision

As models become more accurate the bounds on utility become narrower. These narrower utility bounds tend to decrease the opportunity cost of the decision made under imprecision. However, more accurate models cost more, so a trade-off exists. To make systematic decisions about this trade-off, the DM needs some knowledge about the net value of model alternatives. We assume that the cost and accuracy of a model is known and show that this information allows us to bound the net value of more accurate models, as explained in the next section.

4.2 Bounding the Value of Models

In general, the value of a more accurate model is the change in realized utility due to the decision change caused by using the more accurate model. The bounds on the value of a more accurate model, $\left[\underline{V},\overline{V}\right]$, are a function of the output of the coarse model, the DM's decision policy, the accuracy of the more detailed model, the DM's utility function, and the actual output of the more accurate model:

$$\left[\underline{V},\overline{V}\right]=f\begin{pmatrix}\text{prior knowlege, decision policy,}\\ \text{model accuracy, utility function, model output}\end{pmatrix}. \tag{32}$$

As more of these factors are taken into account in the computations, the value bounds become tighter, but the derivation of such bounds becomes increasingly complex.

In this section, an approach for bounding the value of more accurate models is illustrated for four cases. We start with the case of measuring the value of a model after it has been used, since this is the simplest case and provides the reader with an intuitive understanding of value bounding. Next, we determine the bounds on the value of a model that outputs the precise utility function. This case is helpful because it identifies the maximum gain in value that can be achieved by using more accurate models. Neither of these cases can be used to guide model selection because the first requires that the DM use the model before determining its value and the second relies on perfect models, something that does not exist in engineering design. The third case predicts the value bounds of more accurate models considering the Hurwicz criterion (Arrow and Hurwicz 1972; French 1988). The fourth and final case incorporates the utility function and accuracy of the model into the value bounds computation for the special case of the maxi-min decision policy. This fourth case appears to be the most useful for guidance of the model selection decision and is explored in the design example.

In all of the cases, we focus on the scenario in which a less accurate model has already been used in the analysis. In this scenario, the DM must either decide to make a decision under the current level of imprecision or to acquire and use a more accurate model.

4.2.1 The value of a model after it has been used

If the DM decides to acquire and use a more accurate model he or she now has a set of refined utility bounds from which a new decision can be made. By making a new decision, the DM has selected some new interval of utility that can be achieved, as shown in Figure 20.

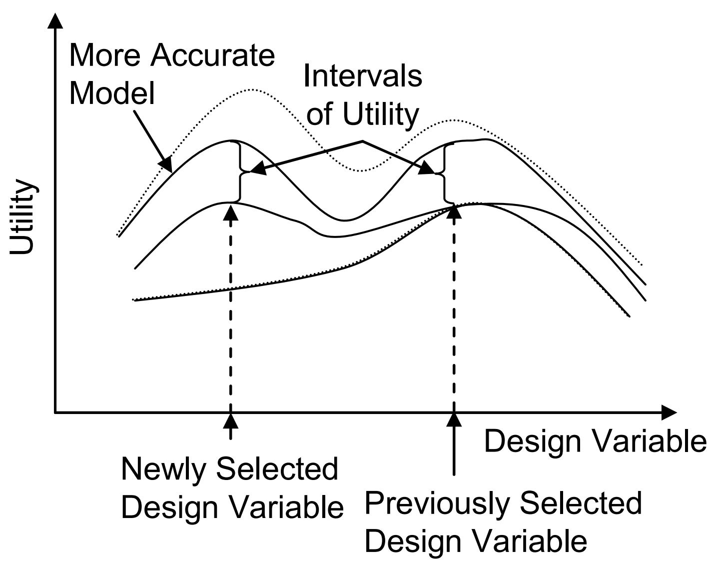

Figure 20: Updated intervals of utility after a more accurate model has been used

The DM can bound the value of the more accurate model based on the two intervals depicted in Figure 20. For clarity, the intervals of utility have been isolated in Figure 21 and the decision before and after using the more detailed model have been labeled as d_0

and d_1, respectively. These intervals represent the possible range of utility for the two design decisions.

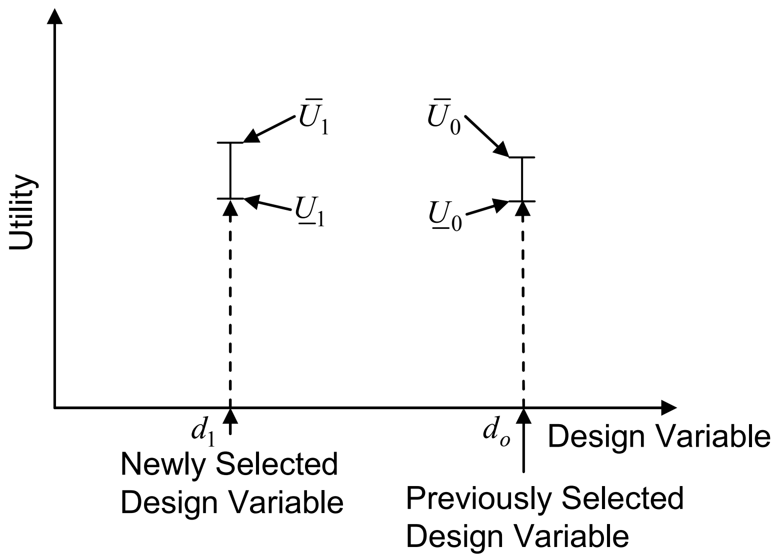

Figure 21: Two utility intervals after using a more accurate model

The bounds on the net value of a more accurate model after it has been used are

$$\left[\underline{V}_{post}, \overline{V}_{post}\right] = \left[\underline{U}_1 - \overline{U}_0, \overline{U}_1 - \underline{U}_0\right]. \tag{33}$$

These bounds can be derived from the two intervals shown in Figure 21. The maximum gain in utility due to the change in decision occurs when the lowest possible utility, $\underline{U}_0$, would have been realized for the previous decision d_0 and the highest possible utility, $\overline{U}_1$, is realized for the new decision d_1, hence $\overline{V}_{post} = \overline{U}_1 - \underline{U}_0$. The maximum possible loss in utility is the difference between the minimum utility that could be realized given the new decision, $\underline{U}_1$, and the maximum utility that could have been realized given the previous decision, $\overline{U}_0$, hence $\underline{V}_{post} = \underline{U}_1 - \overline{U}_0$.

4.2.2 Value of a perfect model

Now that the bounds for an already used model have been derived, we move to the simplest case of *predicting* the bounds on the value of a model, the bounds on a perfect model. A perfect model outputs the precise performance over the feasible design space which can be mapped through the utility function to precise utilities. The upper bound on the net value of a perfect model is equivalent to the maximum opportunity cost for making a decision under the current level of imprecision. If the DM cannot tolerate this opportunity cost, then he or she should acquire a more accurate model.

Lower Bound

The lower bound on the gross value of a perfect model is zero. The lower bound on the value of a perfect model is the most the DM can lose by gaining precise knowledge of the utilities corresponding to the design alternatives. Remember the case in which a decision is made under imprecision as shown in Figure 18. In this case, being given precise utilities can either change the DM's decision or not. If the DM makes the optimal decision based on imprecision, then gaining precise utilities will not change the DM's decision. In this case, the realized utility of the design remains unchanged and precise utility information has zero value. For the other case, in which gaining precise utilities does change the DM's decision, the DM must be changing from a non-optimal prior decision to the optimal decision. This information leads the DM to a decision yielding higher utility and therefore has positive value. In conclusion, the lower bound of the value of a perfect model is zero.

Upper Bound

The maximum value of a perfect model, $\overline{V}_p$, is the difference between the worst utility that can be realized given the current decision under imprecision, $\underline{U}_0$, and the highest utility that could be realized, $\overline{U}_{\max}$: $\overline{V}_p = \overline{U}_{\max} - \underline{U}_0$. The worst utility that can be realized given the DM's decision under imprecision is the lower bound on the utility interval for that decision, $\underline{U}_0$ in Figure 17. The highest utility that could be realized is the maximum of the upper bound on the imprecise utility function over the domain of interest, $\overline{U}_{\max}$ in Figure 18.

For example, the decision and precise utility curve shown in Figure 22 yields the upper bound on the value of a perfect model. The true utility curve shows that the utility of the selected design decision is the worst possible $\underline{U}_0$, while the optimal decision yields the maximum possible utility, $\overline{U}_{\max}$.

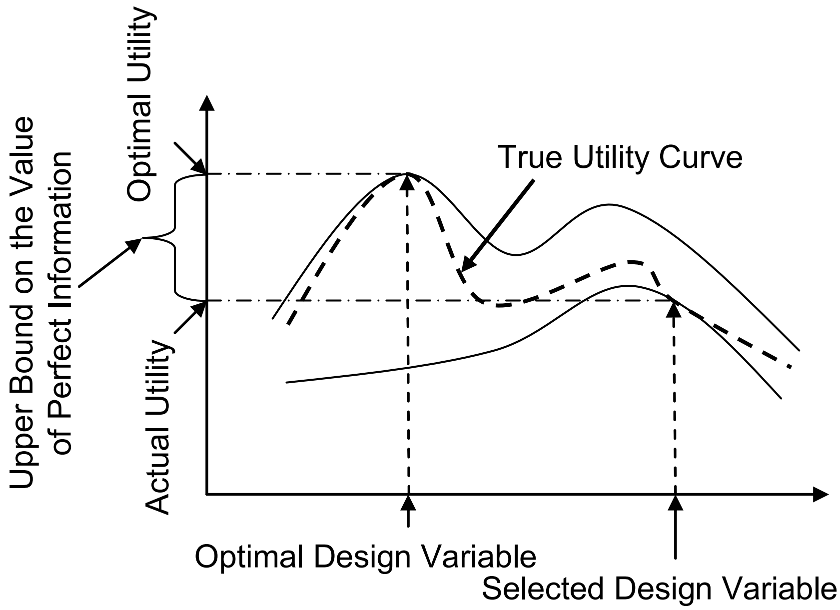

Figure 22: The upper bound on the value of a perfect model

In summary, the bounds on the value of a perfect model are:

$$\left[\underline{V}_p, \overline{V}_p\right] = \left[0, \overline{U}_{\max} - \underline{U}_0\right]. \tag{34}$$

Although conceptually useful, the value of a perfect model alone cannot guide model selection because perfect models do not exist in engineering design practice.

4.2.3 Incorporating Hurwicz decision policy

Previously, we derived value bounds based solely on prior knowledge in the form of existing bounds on performance. Now, we add the DM's selection of the Hurwicz decision policy to the value bounds derivation. Then, we derive the value bounds for a special case of the Hurwicz decision policy (the maxi-min decision policy), which allows model accuracy and the utility function to be incorporated. This final derivation is explored with an example design problem.

The Hurwicz decision policy is explained in Section 2.4. The Hurwicz decision policy specifies a decision point, U^{α}, in the interval of utility according to the optimism-pessimism index, $0 \le \alpha \le 1$, and the equation $U^{\alpha} = \alpha \underline{U} + (1-\alpha)\bar{U}$. When incorporating the Hurwicz decision policy into the value bounds derivation, there are three possible cases that must be considered, listed below and shown in Figure 23:

Case 1) both decision points lie outside the intersection of the utility intervals and the intervals overlap,

Case 2) one or both decision points are contained in the intersection of the utility intervals, and

Case 3) one interval dominates the other interval. If the utility intervals do not overlap, the gross value of all more accurate models is zero (i.e., the DM should not consider using a more accurate model).

In this section, we do not make any assumptions about the size of the utility interval that will result from using a more accurate model. The analysis presented here assumes a utility interval that results in a worst-case scenario — the largest possible upper bound on the value and the most negative lower bound.

<table>
<tr><td colspan="2"></td><td>$\alpha < 0.5$</td><td>$\alpha > 0.5$</td></tr>
<tr><td>Case 1</td><td>$U_1^{\alpha} < \underline{U}_0$
and
$U_0^{\alpha} > \overline{U}_1$</td><td>✓</td><td>✓</td></tr>
<tr><td>Case 2</td><td>$U_0^{\alpha} < \overline{U}_1$
and, or
$U_1^{\alpha} > \underline{U}_0$</td><td>✓</td><td>✓</td></tr>
<tr><td>Case 3</td><td>$\underline{U}_0 > \overline{U}_1$</td><td colspan="2">✓</td></tr>
</table>

Figure 23: A list of the possible cases for bounding the value of a more accurate model

For Case 1 and Case 2, we must consider the optimism-pessimism index in two parts $\alpha \geq 0.5$ and $\alpha < 0.5$ because the solution changes at $\alpha = 0.5$, as explained below.

Regardless of the value of α, the upper bound of the value of a more accurate model is always the same. The possibility always exists that the lower bound, $\underline{U}_0$, on the utility interval for the current decision is realized while the best possible utility, $\overline{U}_1$, for the alternative not selected turns out to correspond to reality. Hence, the upper bound on value is: $\overline{V} = \overline{U}_1 - \underline{U}_0$.

The lower bound on value is dependent on the utility bounds predicted by the coarse model. Suppose that the coarse model yielded the utility bounds shown in Figure 24. From the previous derivation, the lower bound on the value of a model after it has been used is $\underline{V}_{post} = \underline{U}_1 - \overline{U}_0$. This formula is only valid for cases in which the decision changed from design variable d_0 to design variable d_1 due to the use of the model. To find the lower bound on the value of a more accurate model before it is used, we must find $\underline{V} = \min\left(\underline{V}_{post}\right)$, the minimum lower bound for any more accurate model, given that the decision changes.

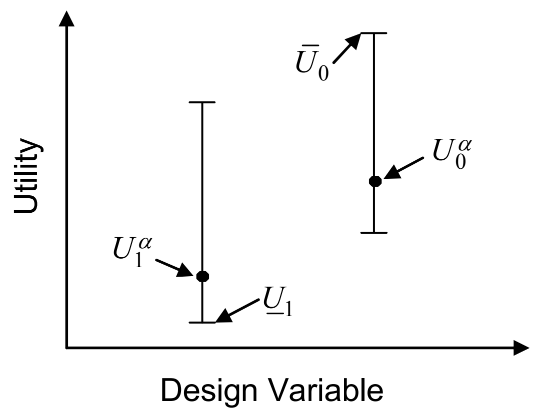

Figure 24: Utility intervals for two design alternatives

Based on our assumptions, the more accurate model can output any reduced intervals of utility for the two design alternatives, as long as those intervals are contained within the utility intervals predicted by the coarse model. A possible set of reduced utility intervals based on the utility intervals shown in Figure 24 are denoted with solid lines and primes in Figure 25. To find $\underline{V} = \min\left(\underline{V}_{post}\right)$, we must isolate the set of reduced utility bounds that yield $\underline{V}$. In other words, the utility bounds that maximize $\bar{U}_0' - \underline{U}_1'$ (i.e., minimize $\underline{U}_1' - \bar{U}_0'$), while insuring that the decision changes, $U_1'^{\alpha} > U_0'^{\alpha}$, where primes denote output from the more accurate model for $\alpha \geq 0.5$. In the following sections, we isolate the set of new utility bounds that yield $\underline{V} = \min\left(\underline{V}_{post}\right)$ for the cases specified in Figure 23.

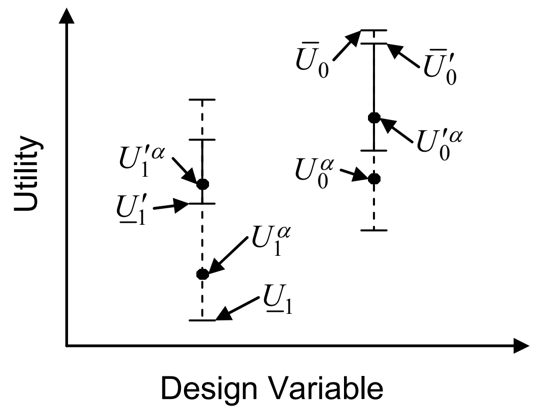

Figure 25: A possible set of reduced utility intervals (shown as the solid line intervals)

Case 1

In case one, both decision points, U_0^{α} and U_1^{α}, lie outside the intersection of the two utility intervals based on the output of the coarse model, as shown in Figure 26 for $\alpha \geq 0.5$. We start by deriving the lower bound on value of a more accurate model for the case in which $\alpha \geq 0.5$ and then similar results for $\alpha < 0.5$ are summarized.

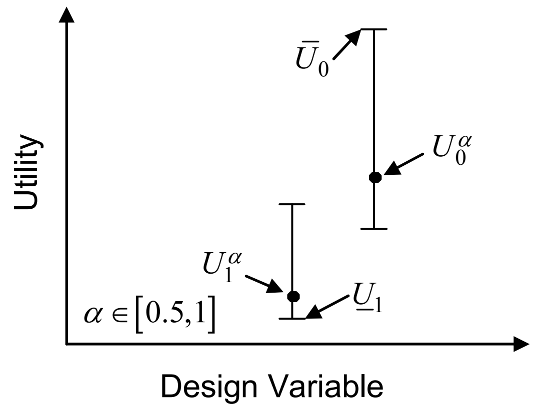

Figure 26: A set of possible utility intervals $\alpha \geq 0.5$

To isolate the set of utility bounds that yield $\underline{V}$, we must find the set of reduced utility bounds that maximize $\bar{U}_0' - \underline{U}_1'$, while satisfying $U_1'^{\alpha} > U_0'^{\alpha}$. We isolate this set through a constructive proof in which we start by assuming that the more accurate model outputs the same utility intervals as the coarse model, see Figure 27. We then shift the bounds on the utility intervals of the more accurate model as necessary to find $\underline{V}$. We can shift the utility bounds for the more accurate model as long as the new set of utility bounds is subsumed by the utility bounds derived from the coarse model.

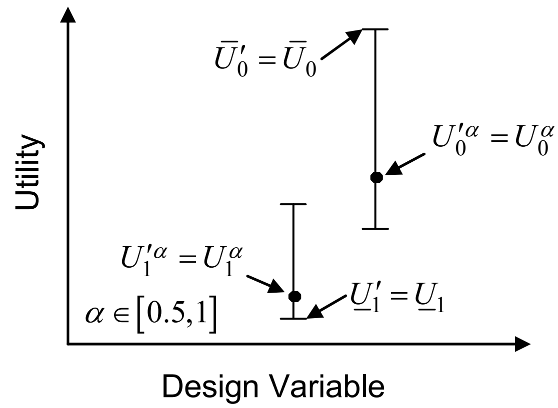

Figure 27: The starting point for the value bounds derivation for which the utility interval from the more accurate model (denoted with primes) is equal to the utility interval from the coarse model

To satisfy $U_1'^{\alpha} > U_0'^{\alpha}$, the decision points must first be brought together, since initially $U_0'^{\alpha} > U_1'^{\alpha}$; this can be accomplished by either shifting $\underline{U}_1'$ up or shifting $\bar{U}_0'$ down. Shifting either of these bounds directly reduces the quantity that we are trying to maximize $\bar{U}_0' - \underline{U}_1'$, so we want to shift them as little as possible while satisfying $U_1'^{\alpha} > U_0'^{\alpha}$. Shifting $\underline{U}_1'$ up by one util (a unit of utility), moves the decision points α utils closer, while shifting $\bar{U}_0'$ down one util moves the decision points $(1-\alpha)$ utils closer. For $\alpha \geq 0.5$, we have that $\alpha > (1-\alpha)$ so we shift $\underline{U}_1'$ up until either the decision is changed or the interval is reduced to a scalar, an interval of size zero. For case 1, the decision cannot be changed by only increasing $\underline{U}_1'$, so the interval of utility for decision 1 is reduced to the scalar $\underline{U}_1' = U_1'^{\alpha}$ as shown in Figure 28.

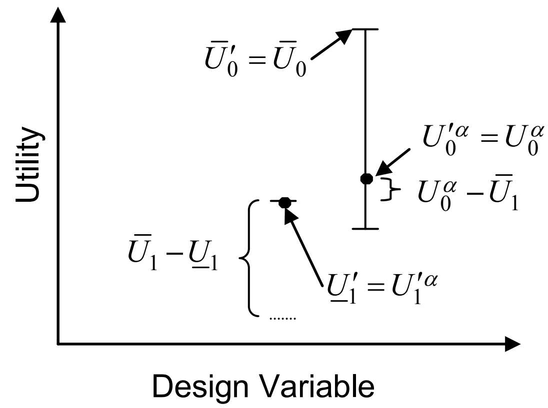

Figure 28: Reducing the utility interval for decision 1 to a scalar

The unaltered decision point, $U_0'^{\alpha}$, is now shifted down the distance $U_0^{\alpha} - \bar{U}_1$, making the decision change, i.e., satisfying $U_1'^{\alpha} > U_0'^{\alpha}$. To do accomplish this shift, $\bar{U}_0'$ must be moved $\left(U_0^{\alpha} - \bar{U}_1\right) / \left(1-\alpha\right)$ utils downward, as shown in Figure 29.

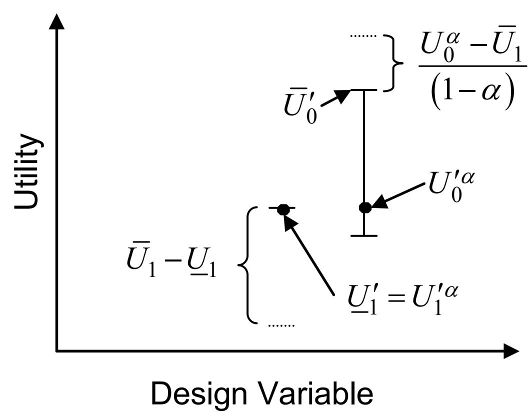

Figure 29: The scenario that yields the lower bound of value for Case 1 - $\alpha \geq 0.5$

At this point, the decision has changed and the lower bound on the value, $\underline{V}$, of the more accurate model for $\alpha \geq 0.5$ follows:

$$\underline{V} = \underline{U}_1' - \bar{U}_0' = \underline{U}_1 - \bar{U}_0 + \left(\bar{U}_1 - \underline{U}_1\right) + \frac{\left(U_0^{\alpha} - \bar{U}_1\right)}{\left(1-\alpha\right)} \tag{35}$$

$$\underline{V} = \bar{U}_1 - \bar{U}_0 + \frac{\left(U_0^{\alpha} - \bar{U}_1\right)}{\left(1-\alpha\right)}. \tag{36}$$

For $\alpha < 0.5$, the same solution methodology applies except the bounds on utility are reduced in the opposite order since $\left(1-\alpha\right) > \alpha$, see Figure 31. First, $\bar{U}_0''$ is shifted down until the interval becomes a scalar, then $\underline{U}_1''$ is shifted up until the decision is changed (where double prime denotes the output of the more accurate model for $\alpha < 0.5$).

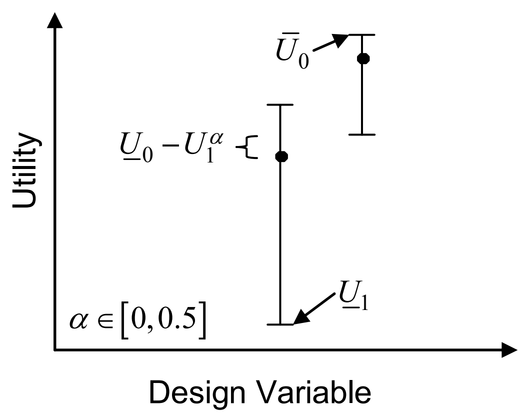

Figure 30: A set of possible utility intervals for Case 1 - $\alpha < 0.5$

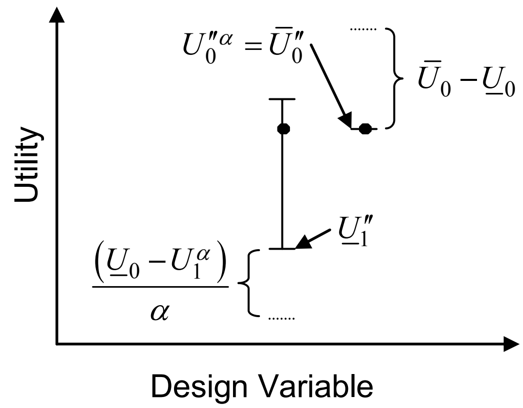

Figure 31: The scenario that yields the lower bound of value for Case 1 - $\alpha < 0.5$

For $\alpha < 0.5$, the lower bound on the value of the more accurate model is:

$$\underline{V} = \underline{U}_1'' - \bar{U}_0'' = \underline{U}_1 - \bar{U}_0 + \left(\bar{U}_0 - \underline{U}_0\right) + \frac{\left(\underline{U}_0 - U_1^{\alpha}\right)}{\alpha}. \tag{37}$$

$$\underline{V} = \underline{U}_1 - \underline{U}_0 + \frac{\left(\underline{U}_0 - U_1^{\alpha}\right)}{\alpha} \tag{38}$$

Case 2

In case two, one or both of the decision points, U_0^{α} and U_1^{α}, lie in the intersection of the two utility intervals, see Figure 32. Instead of shifting both $\underline{U}_1'$ and $\bar{U}_0'$ to find the lower bound on the value of a more accurate model, Case 2 requires only one shift, a shifting of the bound that has the most effect on the decision point, as shown in Figure 32 and Figure 33 for $\alpha < 0.5$.

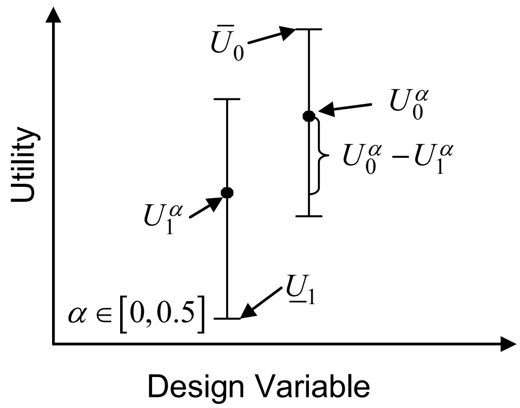

Figure 32: A set of possible utility intervals for Case 2 - $\alpha < 0.5$

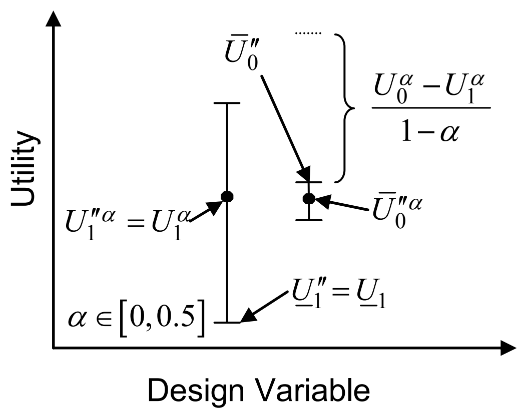

Figure 33: The scenario that yields the lower bound of value for Case 2 - $\alpha < 0.5$

The lower bounds on the value of a more accurate model for Case 2 are:

For $\alpha \geq 0.5$:

$$\underline{V} = \underline{U}_1' - \bar{U}_0' = \underline{U}_1 - \bar{U}_0 + \frac{U_0^\alpha - U_1^\alpha}{\alpha}. \tag{39}$$

For $\alpha < 0.5$:

$$\underline{V} = \underline{U}_1'' - \bar{U}_0'' = \underline{U}_1 - \bar{U}_0 + \frac{U_0^\alpha - U_1^\alpha}{(1-\alpha)}. \tag{40}$$

Solving equation (39) for the maximin decision policy, $\alpha = 1$, yields a simplified formula:

$$\underline{V} = \underline{U}_1 - \overline{U}_0 + \frac{U_0^\alpha - U_1^\alpha}{\alpha} = \underline{U}_1 - \overline{U}_0 + U_0^\alpha - U_1^\alpha = \underline{U}_1 - \overline{U}_0 + \underline{U}_0 - \underline{U}_1 . \tag{41}$$

$$\underline{V} = \underline{U}_0 - \overline{U}_0 \tag{42}$$

Using the maxi-min decision policy, the lower bound on the value of more accurate models only depends on the utility interval of the initial decision alternative. In more detail, selection of the maxi-min decision policy implies that the lower bound on the utility of the new decision is bounded below by the lower bound on the current decision because the lowest bound on any future decision, must be higher than the lower bound of the current decision for it to become the preferred alternative. Hence, the case that maximizes the difference of interest, $\underline{V} = \underline{U}_1 - \overline{U}_0$, occurs when the lower bound on the new decision is equal to the lower bound of the current decision, $\underline{U}_1 = \underline{U}_0$. Use of the maxi-min decision policy is explored in detail in the next section and is the focus of the design example.

4.2.4 Incorporating accuracy and utility

In the previous sections, the value bounds derivation was based only on the assumption that the model accuracy statements are valid over the domain of interest (#2 Key assumptions below). The bounds on value derived from these analyses were bounds on any more accurate model, so they could help the DM decide if a more accurate model my be valuable, but not which more accurate model to choose.

In this section, we develop an approach that allows us to bound the value of a particular model by expanding the results of the previous section specifically for the maxi-min decision policy. Use of the maximin design decision policy allows us to present the approach in a simple form that focuses on the model selection decision. Additionally, we assume that we know the cost and accuracy of each of the model alternatives, which are essential criteria for selection. A summary of these assumptions is provided in the Key assumptions box below.

Key assumptions

1. The cost and accuracy of the more accurate model alternatives are known.
2. Model accuracy statements are valid over the domain of interest.
3. The design decision is based on a maxi-min decision criterion $(\alpha = 1)$.

Once the DM has selected the maximin design decision policy, he or she can derive the bounds on the net value of a particular more accurate model using the bounds on performance given by the coarse model, knowledge about the accuracy of a prospective model, and his or her utility function, as shown in Figure 34. In the method proposed, we derive value bounds for a set of design alternatives since we do not know d_1, the decision alternative that will be selected after the more accurate model is used. In contrast, the previous value bounding analyses only considered bounds on utility for two design alternatives, because we assumed knowledge of d_1. By selecting two design variable alternatives and the corresponding utility intervals from the top-right box of Figure 34, we return to the previous value bounding cases. We now move to the derivation of the value bounds of a particular more accurate model.

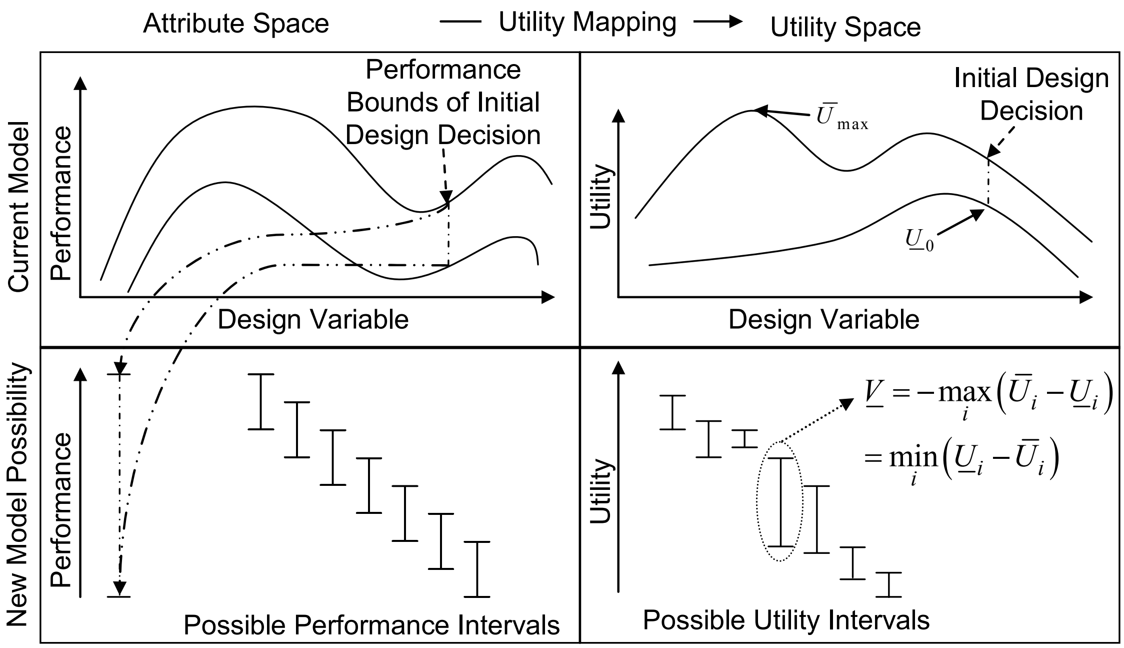

Figure 34: An approach for determining the lower bound on value

To find the lower bound on the value of a particular more accurate model, the DM begins by mapping the known performance bounds to utility bounds using his or her utility function, Figure 34 top. With bounds on utility and a selected decision criterion, the DM makes the initial design decision. This initial design decision corresponds to a preferred interval of performance. This interval of performance is discretized into m new intervals that have width based on the accuracy of the new model. These discretized performance bounds are mapped through the utility function to derive a set of m possible utility bounds $\left[\underline{U}_i, \bar{U}_i\right]_{i=1..m}$, Figure 34 bottom. The lower bound on value is:

$\underline{V} = -\max_i \left(\bar{U}_i - \underline{U}_i \right) = \min_i \left(\underline{U}_i - \bar{U}_i \right)$. For simplicity we explain the approach for finding $\underline{V} = -\max_i \left(\bar{U}_i - \underline{U}_i \right) = \min_i \left(\underline{U}_i - \bar{U}_i \right)$ using an exhaustive discretization, but optimization should be used in more complex problems.

4.2.4.1 Decision Policy selection

Once the lower bound on the gross value interval for each possible more accurate model is determined and the cost of the model subtracted to arrive at net value, it is time for the model selection decision. Model selection requires a decision under imprecision in which the intervals of net value of all model choices either contain zero or are entirely negative, see Figure 35. In the example problem, we chose the $\alpha = 0.5$ Hurwicz decision criterion for use in the model selection decision. The $\alpha = 0.5$ decision policy allows the most preferred model to be selected, as shown in Figure 35. Once a model is selected, it is acquired and used and the model selection process can be repeated with the new model serving as the coarse model.

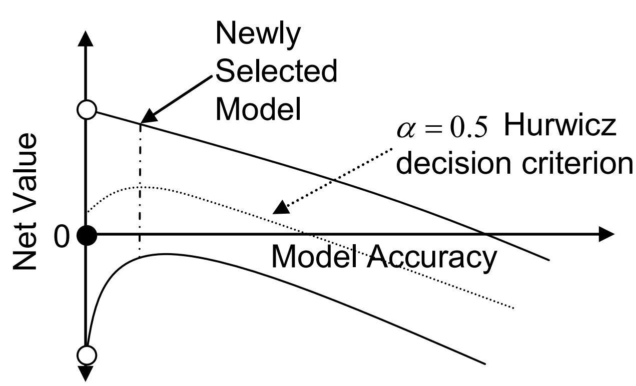

Figure 35: A plot of net value vs. model accuracy

4.2.4.2 An approach for model selection

In the previous sections, we have described an approach for bounding the value of using a more accurate model considering the output from the coarse model, the design decision policy, the accuracy of the model, and the utility function of the DM. This section summarizes the steps required for applying this value bounding approach.

Making the original design decision:

1. Define the DM's preference function over the design variable space.
2. Select an initial coarse model to begin the analysis.
3. Use the output of the initial model to bound the performance over the design decision space. (See Figure 34 top-left)
4. Map the performance space to the utility space and select the preferred design alternative based on the maxi-min decision policy. (See Figure 34 top-right)

Bounding the value of more accurate models:

5. Calculate the upper bound on the value of more accurate models. The upper bound is the difference between the highest payoff obtainable, based on the output of the coarse model, and the lower bound on the payoff of the current decision, $\overline{V} = \left(\overline{U}_{\max} - \underline{U}_0\right)$, Figure 34 top-right.
6. Determine the cost and accuracy of other possible models. We assume such knowledge is readily available. For instance, the DM is purchasing the model from a vendor and such information is provided by the vendor.
7. Determine the lower bound on the value of each more accurate model.

a. Find the performance bounds for the initial design decision selected in Step 4. (See Figure 34 top-left)

b. Determine the possible performance intervals for the more accurate model based on the model's accuracy and the limiting bounds on performance. (See Figure 34 bottom-left)

c. Map each of the possible performance bounds to the utility space. (See Figure 34 bottom-right)

d. The lower bound on value that could be achieved (i.e. the most that could be lost) by using the model is $\underline{V} = -\max_i \left(\overline{U}_i - \underline{U}_i \right) = \min_i \left(\underline{U}_i - \overline{U}_i \right)$. (See Figure 34 bottom-right)

Note: In steps b. through d. $\underline{V}$ is found by discretizing the possible performance intervals to make the presentation of the approach clear, but optimization should be used to find $\underline{V}$ for more complex problems.

8. Subtract the model's cost from the upper and the lower bound on the value to arrive at the upper and lower bounds on the net value. (See Figure 35)
9. Return to step 7b. until the net value bounds for each possible model are known.
10. Select the most desired model based on the net value bounds and the $\alpha = 0.5$ Hurwicz decision criterion. (See Figure 35)
11. If the chosen model is the current model, stop; else, acquire and use the model selected in Step 8. Return to Step 2. This new model becomes the new initial model.

4.3 Example Problem

In this section, the approach for bounding the value of a more accurate model is illustrated with an I-beam structure design adapted from (Hoff 1956). A description of the design scenario and the computational experiment follows.

4.3.1 Design Scenario

The DM needs to design an I-beam structure to resist uniform loading, while satisfying deflection requirements, see Figure 36. The goal is to determine the thickness, t, of the I-beam flanges and web that maximizes utility, see Figure 36. It is assumed that the engineering code book set a deflection limit of 0.0089m (0.35in) on this structure.

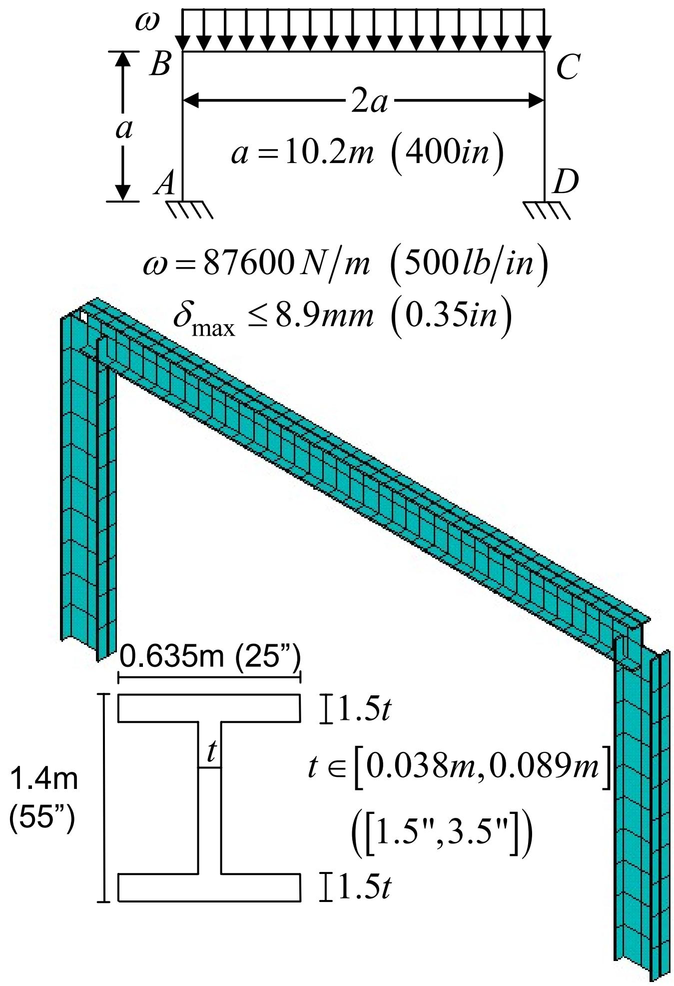

Figure 36: Design scenario: geometry and loading

Since the DM wishes to satisfy deflection constraints while minimizing the cost of material, he or she specified the following utility function:

$$
\begin{aligned}
&Utility = Sellprice - Cost \qquad (43)\\
&where:\\
&Sellprice = \$40,000 \cdot \frac{1}{1+e^{1180(\delta - 0.0089)}}.\\
&\delta = deflection\ (m)\\
&Cost = UnitCost \cdot VolumeOfMaterial\\
&UnitCost = \$5120/m^3\ \ (\$145/ft^3)
\end{aligned}
$$

A sigmoid function was used to approximate the price at which the DM could sell the I-beam structure, see Figure 37. This function was chosen for the example problem because we believe it is a reasonable approximation of the relationship between sell price and maximum deflection.

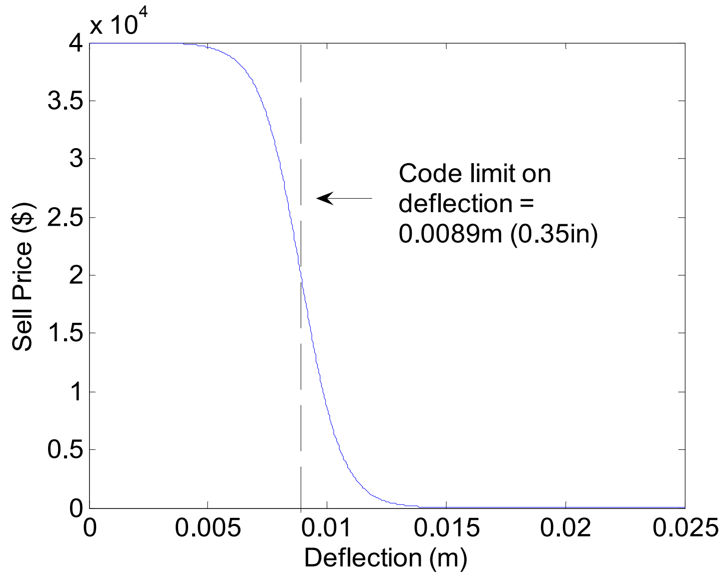

Figure 37: Deflection value function used in example problem

We assume the DM does not know the exact deflection that will result for thicknesses, t; instead, he or she only has bounds on the deflection, which are derived from the model prediction and the maximum error of the model. The designer therefore must not only make the design decision by specifying the thickness, t, but also must make the model selection decision by deciding how accurate of a model to use in the specification of t. The model selection decision is the focus of the computational experiment.

4.3.2 Computational Experiment

Ansys, a finite element analysis software, was used to construct the models of the I-beam structure that were used in the model selection decision, Figure 36. Similar to the derivation in (Hoff 1956), assuming pure bending, the maximum deflection in the I-beam structure is $\delta_{\max} = \dfrac{\omega a^4}{12EI}$, where I is the moment of inertia of the cross section. The Ansys model converges to a slightly different maximum deflection because element types 188 and 189 are used; both element types consider shear deformation effects in addition to pure bending. For example, with a thickness of 0.043m (1.68in), the predicted maximum deflection is 0.0090m (0.353in), while Ansys converges to a solution of 0.0097m (0.382in). In general an analytical solution is not available for more complex systems. Consequently, we assume that the DM has no knowledge of the analytical solution.

The experiment was conducted as follows. The number of finite elements used in the models ranged from 6 to 192 ($6 \cdot 2^5$). For each model the maximum deflection prediction was computed for thicknesses between 0.038m and 0.089m (1.5in and 3.5in). The maximum deflection prediction and CPU time were recorded also for each model. The deflection predictions for a model with 384 ($6 \cdot 2^6$) elements was assumed to be the truth. The difference in the predictions between the varying models and the 384 element model was assumed to be the error for that particular model. The maximum percent error taken over the design range was recorded for each model and is shown in Figure 38. Figure 39 shows the average computational cost for each model measured in seconds of cpu time.

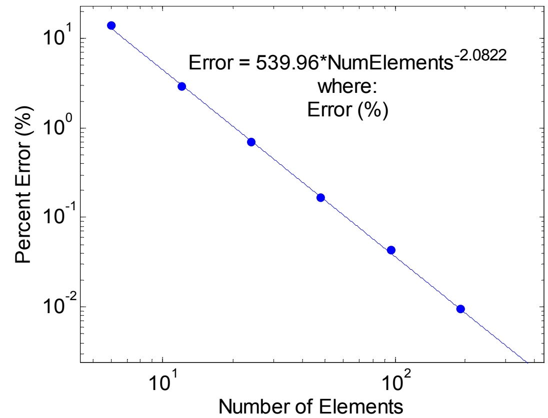

Figure 38: Error vs. number of elements

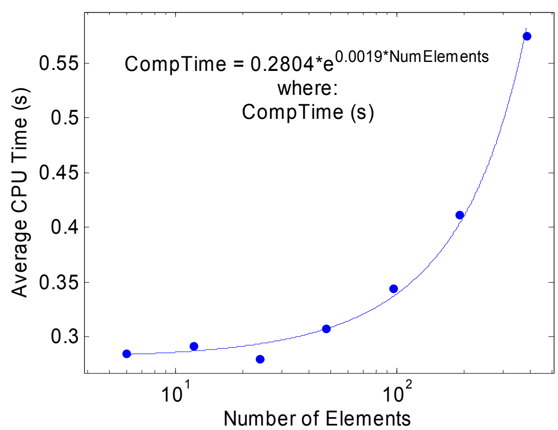

Figure 39: Computational time vs. number of elements (computational time is considered a proxy for total model cost according to the equation $ModelCost = CompTime \cdot \$1000/s$ **)**

For the purpose of this example, the cost of acquiring and using the model is assumed proportional to computational time. Specifically, $ModelCost = CompTime \cdot \$1000/s$.

Now that the cost and accuracy of the finite element models are known for varying numbers of elements, the approach for model selection can be applied to find the most preferred number of elements. The outcome of the application of the approach is summarized in the following section.

4.4 Explanation of results

In this section, we step through applying the approach for model selection that is given on page 82 in section 0 to the example problem. We then discuss the results that are derived from application of the approach.

4.4.1 A walkthrough of the approach for model selection

Step 1: Define the DM's preferences.

This step was completed when we defined the DM's utility function, Equation (43).

Step 2: Select an initial model.

We assume that the DM chooses to use an initial coarse model of 12 elements.

Step 3: Determine performance bounds.

The performance in this example is expressed in terms of the maximum deflection in the structure. The performance bounds predicted by the twelve element model are shown in Figure 40.

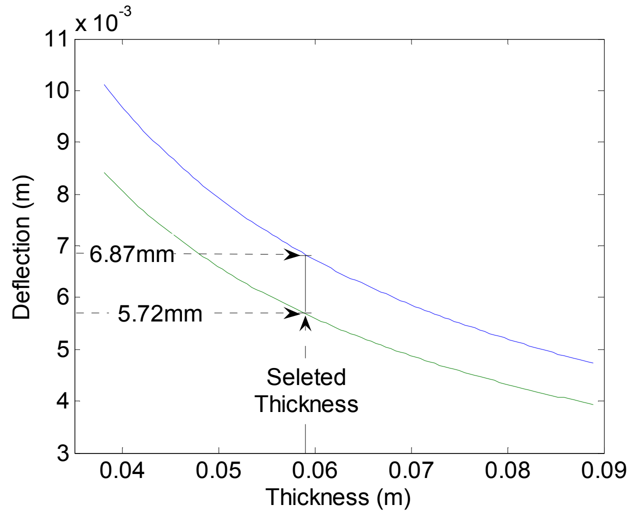

Figure 40: The deflection bounds based on the output of a twelve element model (a thickness of 0.059m was selected based on the maximin decision policy as shown in Figure 41)

Step 4: Select a design alternative based on the maximin design decision policy.

The first part of step 4 entails mapping the deflection bounds from Figure 40 through the DM's utility function to bounds on utility, shown in Figure 41. Based on these utility bounds and the maximin decision policy, the DM selects a thickness of 0.059 m, which guarantees the DM of a design with utility of at least \$7170. Figure 40 and Figure 41 correspond to the top half of Figure 34: An approach for determining the lower bound on value.

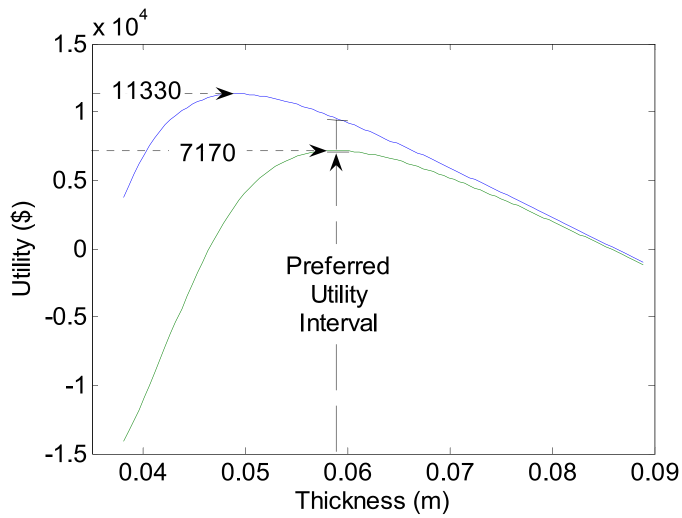

Figure 41: The utility bounds based on the output of a twelve element model, (a thickness of 0.059m was selected based on the maximin decision policy)

Step 5: Calculate the upper bound on the value of more accurate models.

As derived earlier, the upper bound on the value of more accurate models is $\bar{V} = \left(\bar{U}_{\max} - \underline{U}_0\right)$. From Figure 41, we see that $\bar{U}_{\max} = \$11330$ and $\underline{U}_0 = \$7170$; using these values, we find that $\bar{V} = \$4160$.

Step 6: Determine the cost and accuracy of other possible models.

To simplify step 6 for this example problem, we will only consider one possible model alternative, a model with 24 elements. Based on the error curve and computational time curve shown in Figure 38 and Figure 39, the maximum error in the model is 2.17%, while the model cost is $290. Remember that $290 is assumed to be an estimate of the costs of both acquiring and using the 24 element model.

Step 7: Determine the lower bound on the value of each more accurate model.

For this example, the DM needs to determine the lower bound on the value, $\underline{V}$, of the 24 element model. The procedure for finding $\underline{V}$ is summarized in Figure 34. First, the DM finds the performance (deflection) bounds for the design decision selected in Step 4, top-left in Figure 34. These performance bounds are 5.7mm and 6.8mm, as shown in Figure 40. With these performance bounds, the DM determines the interval of possible midpoints for the performance bounds of the more accurate model. This interval is uniformly discretized, the output being a set of possible midpoints for the performance interval output by the more accurate model. Based on the set of midpoints and the model's accuracy a set of performance interval can be constructed, bottom-left in Figure 34. These intervals of performance are mapped through the DM's utility function to utility intervals, as shown in Figure 42.

This process is shown for a set of three possible midpoints in Figure 42. Although three discretizations (midpoints) suffice for illustrating the approach, it should be noted that 1000 discretizations were used to obtain the computational results.

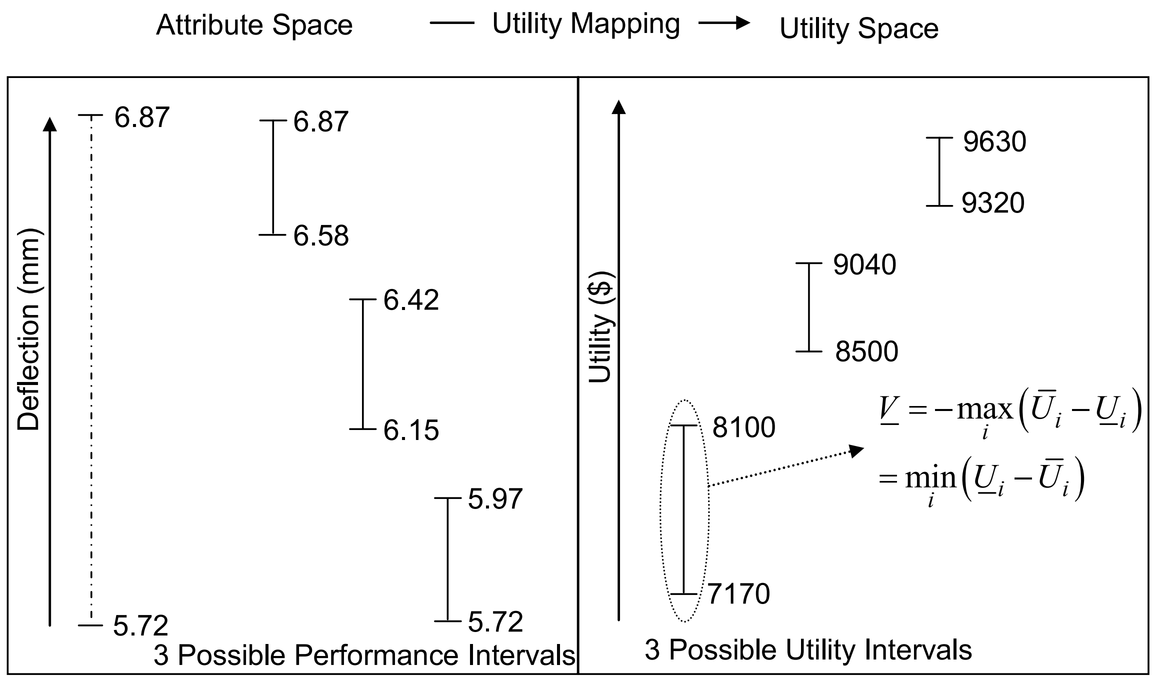

Figure 42: Performance bounds mapped to utility bounds, part of the approach for finding the lower bound on the value of a more accurate model

To complete step 7, the DM finds, $\underline{V} = -\max_i \left(\bar{U}_i - \underline{U}_i \right) = \min_i \left(\underline{U}_i - \bar{U}_i \right)$ the largest interval of utility that could be output by the more accurate model. For this experiment, in which we are only considering three discretization intervals, finding $\underline{V}$ is simple. The interval that yields $\underline{V}$ is circled in Figure 42. The lower bound on value is $\underline{V} = \min_i \left(\underline{U}_i - \bar{U}_i \right) = \$7170 - \$8100 = -\$930$. The DM now knows that the upper bound and lower bound on gross value of the 24 element model is $\left[\underline{V}, \bar{V} \right] = \left[-\$930, \$4160 \right]$.

Step 8: Subtract the cost of the model from the value bounds to find net value bounds.

The cost of the model ($290) is subtracted from the gross value bounds to find the bounds on net value, $\left[\underline{V}_{net}, \bar{V}_{net} \right] = \left[-\$1220, \$3870 \right]$.

Step 9: Find the lower bound on value for the remaining models by returning to step 7.

We save the reader from this repetition and assume that the bounds on net value are computed for all models ranging from 13 to 200 elements. Once the net value bounds are known, they can be plotted, as shown in Figure 43.

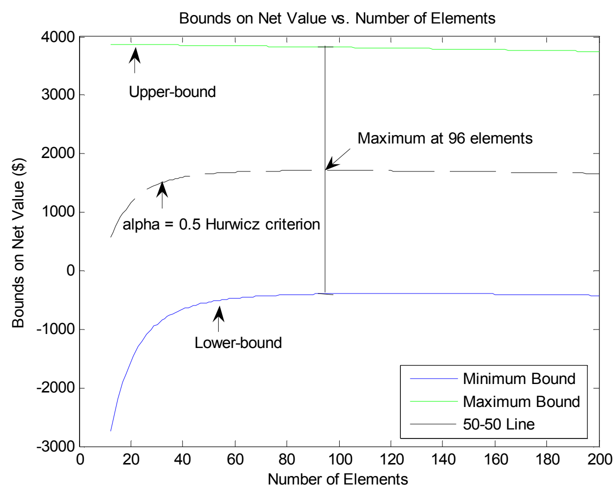

Figure 43: Net value bounds (initial model = 12 elements)

Step 10: Select the most preferred more accurate model.

In this step, the DM uses $U^{\alpha=0.5}$, the utility predicted by the $\alpha = 0.5$ Hurwicz decision criterion, to select a more accurate model. From Figure 43, we see that $U^{\alpha=0.5}$ is maximized for a more accurate model with 96 elements.

Step 11: If the selected model is the current model, stop; else, acquire and use the selected model. Return to step 2; this selected model becomes the new initial model.

According to step 11, the DM acquires and uses a model with 96 elements. The 96 element model now becomes the coarse model and the value bounding analysis is repeated to see if a more accurate model is preferred. We save the reader from such repetition and present the net value bounds that are derived based on a coarse model of 96 elements, shown in Figure 44. From Figure 44, it can be noted that once a model with 96 elements is acquired and used, no more accurate model could have positive net value. For example, developing a model with 97 elements would lead to a loss of at least $281 and possibly a loss as great as $390. Consequently, the DM would stop the analysis and make the design decision based on the 96 element model.

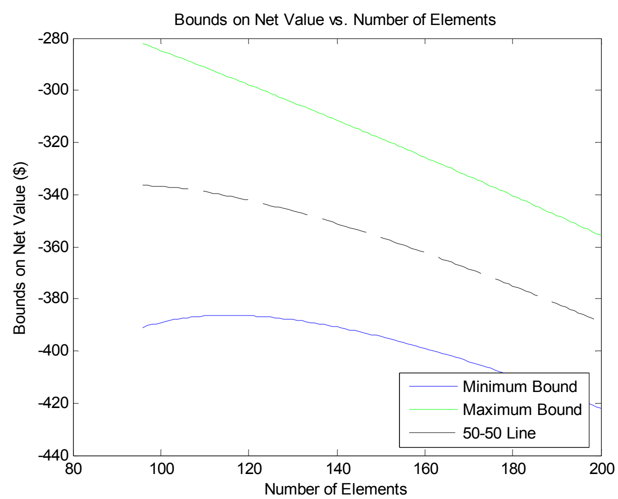

Figure 44: Net value bounds (initial model = 96 elements)

Now that we have walked through the approach, it will be useful to further discuss some aspects of the approach.

4.4.2 The value of the 96 element model after it has been used

In this section, we use the results from section 4.2.1, "The value of a model after it has been used" to compute the gross value bounds of the 96 element model after it has been used. We then subtract the cost of the model from the gross value interval to find the net value bounds. These net value bounds should be a subset of the net value bounds, $\left[\underline{V}_{net}, \overline{V}_{net}\right] = \left[-\$400, \$3820\right]$, shown in Figure 43 because the DM now knows the output form the 96 element model.

Figure 45 and Figure 46 show the performance bounds output by the 12 and 96 element model. Figure 45 is similar to Figure 41 with the addition of the performance bounds output by the 96 element model. Figure 46 shows a larger picture of the zoom area denoted in Figure 45. We provide this alternate view so that the reader can see the similarities between these figures and Figure 21 of section 4.2.1.

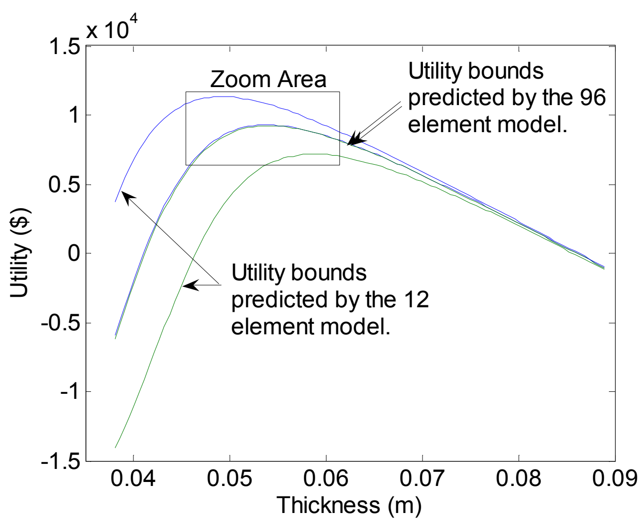

Figure 45: The performance bounds output by the 12 element model and the 96 element model (the zoom area indicated is shown in Figure 46)

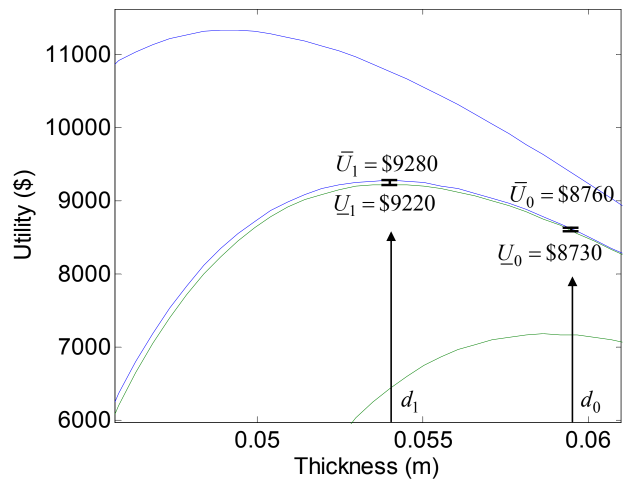

Figure 46: The utility bounds output by the 12 element model and the 96 element model (the utility bounds for the decision based on the twelve element model, d_0, and the decision based on the 96 element model, d_1, are also shown)

The value bounds for the 96 element model after it has been used are computed based on Equation (33): $\left[\underline{V}_{post}, \bar{V}_{post}\right] = \left[\underline{U}_1 - \bar{U}_0, \bar{U}_1 - \underline{U}_0\right]$. Substituting the utility values depicted in Figure 46 into equation (33) yields: $\left[\underline{V}_{post}, \bar{V}_{post}\right] = [\$460, \$550]$. By subtracting the cost of the 96 element model ($290) from this value interval, we arrive at the net value interval for the 96 element model after it has been used: $\left[\underline{V}_{post\_net}, \bar{V}_{post\_net}\right] = [\$170, \$260]$. It is easy to verify that this interval is subsumed by the larger net value interval, $\left[\underline{V}_{net}, \bar{V}_{net}\right] = [-\$400, \$3820]$, that was predicted before the model was used in section 4.4.1. We can attribute the large difference in size between these intervals to the large amount of imprecision in the output of the 12 element model. The difference in the imprecision about the output of the two models can be observed by noting the difference in width of the utility bounds predicted by the two models, as shown in Figure 46.

4.4.3 The lower bound on value is always non-positive

As explained previously, the lower bound on value of all more accurate models is negative, except for the lower bound on the value of a perfect model which is zero. Since the DM cannot obtain a perfect model, the lower bound on the gross value of every more accurate model is negative. In other words, there is always the chance that a more accurate model will cause the DM to change his or her decision to an alternative with a lesser payoff. It should be noted that the possible magnitude of the reduction in payoff decreases as the model accuracy increases.

The one exception to a negative lower bound on value is if the DM chooses not to acquire and use a more accurate model. In this case, he or she cannot gain or lose any money; hence, both the upper and lower bounds on value of this decision are zero, see the solid dot in Figure 35. In both situations, the lower bound on value is non-positive.

4.4.4 Model Selection

In this section, we explore the model selection decision by investigating the net value bounds that are computed based on an initial model of 24 elements. The net value bounds for more accurate models starting from an initial model of 24 elements are shown in Figure 47.

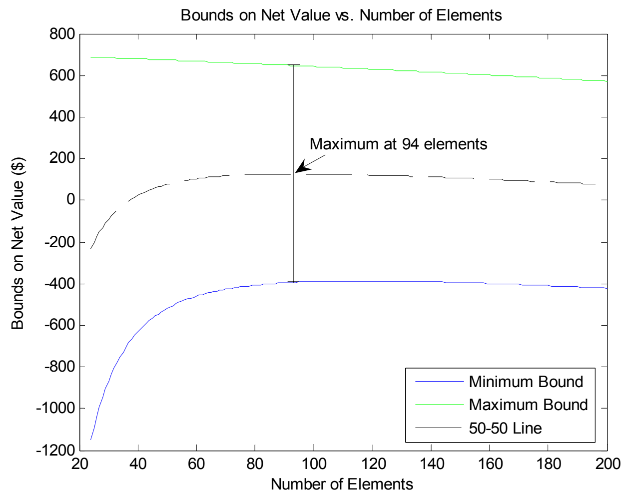

Figure 47: Net value bounds (initial model = 24 elements)

Note the dramatic difference between the value bounds for the most preferred model illustrated Figure 43 and Figure 47. The selected models are almost identical, 96 elements and 94 elements. From Figure 43, the bounds on value of moving from a 12 element model to a 96 element model are [-$400, $3820], while Figure 47 shows that the bounds on value of moving from a 24 element model to a 94 element model are [-$395, $645]. This large difference in the value intervals occurs because there is a decrease in the relative improvement in accuracy from one model to the next as the models become more detailed. Additionally, these increasingly detailed models cost more to acquire and use. Consequently, as the process of acquiring and using more accurate models proceeds, the DM is spending more to acquire and use the suggested models and realizing less improvement for the investment.

Figure 48 serves as a summary of the application of the model selection approach to the design problem. It displays the preferred number of additional elements for the more accurate model over the range of possible initial models. Note that the graph shows the number of additional elements; for example, based on an initial model of 10 elements, the model selection approach suggests an additional 86 elements, implying that the suggested model has 96 elements. Figure 48 does not provide data for initial models with greater than 28 elements because the maximum of $U^{\alpha=0.5}$, the utility predicted by the $\alpha = 0.5$ Hurwicz decision criterion, is negative for coarse models with 29 elements or greater. In such cases, the DM would choose to make the design decision based on the output from the coarse model instead of acquiring and using a more accurate model.

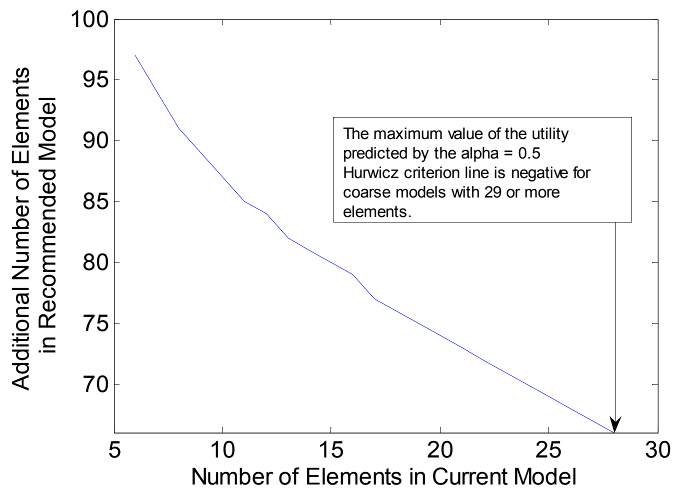

Figure 48: Suggested number of additional elements vs. number of elements in the coarse model

4.4.5 Sensitivity Analysis

This section summarizes the results of a sensitivity analysis performed on several of the design parameters in the example problem. There is no model selection approach that can be compared to this one in an unbiased way. Although, an alternate modeling approach has been suggested (Radhakrishnan and McAdams 2005), they assume possession of a perfect model, which they use as a benchmark for rating other possible model alternatives. They admit that, when using their proposed framework, the designer is left with little guidance in estimating the actual value of different models. The model selection approach proposed here does not assume possession of a perfect model.

It is hoped that support can be built for the soundness of the approach by testing how the model selection decision changes based on changes in the design parameters and evaluating if the changes are intuitive. The results of the sensitivity analysis are summarized in Table 1. The base model used throughout the sensitivity analysis is the 12 element model. The base design problem is the one specified in section 4.3. Notice that the base case in Table 1 corresponds to the selection of a more accurate model of 96 elements, in agreement with Figure 43 in the walkthrough. In the sensitivity analysis, the parameters of the problem were modified by multiplying them by a multiplicative constant. The resultant change in the DM's selection of a more accurate model was analyzed to insure that it agreed with intuition.

Table 1: Sensitivity analysis summary

Parameter	Multiplicative Factor	Num. of Elem. in Model Suggested
Base Model		96
Sell Price	10 1/10	105 75
Material Cost	10 1/10	150 50
Model Cost (Computational)	10 1/10	45 187
Amount of Model Error	3 1/3	130 70

Each of the results in Table 1 appear to agree with intuition. For example, as the price at which the DM can sell the I-beam structure increases, the DM stands to gain more by moving closer to the optimal solution; therefore, the DM should use a more accurate model. In agreement with this, the approach suggests that if the sell price is increased ten fold from the price in the original design problem, the DM will select to use a model with 105 elements instead of 96 elements.

The suggested model is more sensitive to model cost. When the DM uses a more accurate model, he or she is more likely to choose design alternatives closer to the optimum. If all more accurate models cost less, the DM should use a more accurate model for this thickness selection; whereas, if models cost more, the DM should use a less accurate model. The results of applying the model selection approach agree with this. If the models cost 1/10th as much, the approach suggests for the DM to acquire and use a model with 187 elements. If the model cost is increased ten fold, the approach suggests that the DM acquires and uses a model with 45 elements. Similar analyses were performed with material cost and modeling error, as shown in Table 1. These results also agree with our intuition.

It should be noted that this sensitivity analysis is incomplete because only one design parameter is varied from the base model for each case. A complete sensitivity analysis would consider all possible combinations of the parameter variations. However, these limited results are logical; we therefore conclude that the sensitivity analysis supports the applicability of the value bounding approach.

4.5 Summary

DMs inherently lack knowledge during the design process. To make good decisions, the DM must often reduce the amount of imprecision in his or her knowledge by developing and using more accurate models. The main contribution of this chapter is the introduction of an approach for model selection based on bounding the value of more accurate models. We have illustrated the approach for model selection with an I-beam structural design. It was shown that for this problem the model selection approach provided guidance to the DM in selecting the most preferred model. Open questions that remain include: 1) What decision policies are most useful? and 2) How can the approach be applied to more complex problems with multiple decision variables? Additionally, it may be useful to consider the dependency between the models. Currently, the models are assumed nondependent; this assumption is explained in detail in Chapter 5. In Chapter 5, we describe the approach's limitations and identify areas for future work.

5 Discussion and remarks

This chapter concludes this thesis by evaluating the research questions and hypotheses in the context of the work presented, by discussing the contributions and the limitations of this thesis, and finally by providing suggestions of how this thesis might be fruitfully extended in the future.

5.1 Research question summary and hypothesis evaluation

An important part of all academic research is the evaluation of the research questions and validation of the hypotheses. In this section, a summary of each of the two research questions and their corresponding hypotheses is provided. Then arguments in support of the hypotheses are grouped and evaluated. This section concludes that the evidence allows us to accept the hypotheses, that is, that information economic principles can be used to guide decisions about information collection and model selection in the engineering design process.

5.1.1 Research question 1 and hypothesis

Research Question 1: How should a decision maker decide when to stop gathering statistical data when trying to characterize a probability distribution describing a random event?
Hypothesis: The principles of information economics allow the DM to bound the value of the next statistical data point and these bounds can guide the DM towards better decisions about statistical data collection.

Research question 1 asks, "How should a decision maker decide when to stop gathering statistical data when attempting to characterize a probability distribution describing a random event?" It has long been established that if the DM desires a particular confidence level for a hypothesis without considering the cost of information collection, than statistics can guide him or her in the information collection decision. In this thesis, we have argued that the confidence level is not of direct concern to the DM; instead, the DM cares about the *value* that the information is adding to the final design. The realization that value should guide decision making motivated the hypothesis that the principles of information economics could guide the DM towards better decisions about statistical data collection.

In Chapter 3 of this thesis, we derived an information economic approach to making decisions regarding statistical data collection. The chapter started by explaining how information economic principles could be applied to the statistical data collection problem and presented a simple example to illustrate the application of such principles. Then imprecision was incorporated into the approach using the P-box formalism. Incorporating imprecision allowed us to predict the bounds on the value of future information.

The value bounding approach was evaluated with an example pressure vessel design. The approach allowed us to bound the value of the next statistical data sample. Information management decisions were then made based on these bounds. Section 3.6 evaluated the quality of decisions made using the approach. It was shown that in some cases, where the DM luckily got a fairly representative starting set of data points, that the decisions made using the approach were overly conservative. However, when the DM got an initial set of data points with an "unlucky" bias, the decisions using the approach were good. This analysis was performed using precise knowledge about the true probability distribution characterizing the material strength, knowledge that is not available to the DM. Therefore, it was concluded that a designer should keep taking samples until he or she is confident that there is little chance of a large negative payoff. This conclusion supports using the information economic approach proposed in Chapter 3 for statistical data collection.

5.1.2 Research question 2 and hypothesis

Research Question 2: How should a decision maker select the most preferred model given a particular design problem?
Hypothesis: The principles of information economics allow the value of more accurate models to be bounded. The DM can use such bounds to guide model selection decisions.

Research question 2 asks, "How should a DM select the most preferred model given the particular design problem he or she is faced with?" Often, model selection decisions are not made systematically; the designer has several models to choose from and he or she selects one based on his or her past experience or knowledge, group opinion, or some other decision criterion. As for research question 1, we have argued that the DM most cares about the *value* that the chosen model contributes to the design. The realization that value should guide decision making motivated the hypothesis that the principles of information economics can be used to guide the model selection decisions.

In Chapter 4, we derived an information economic approach to making model selection decisions. This chapter starts by explaining how models are used to guide decisions in engineering design. Next, the value bounds for models are derived for four cases, some of which are meant to assist the reader in understanding the value bounding approach while others can be used to practically guide model selection. The final case incorporated the accuracy of the model, the DM's utility function, the current state of knowledge, and the design decision policy for the special case of the maximin design decision policy. This final case was augmented with information economic principles to form an approach for model selection.

This effectiveness of the information economic approach for model selection was evaluated with an I-beam structure design problem. The example showed that the approach allowed the value of more accurate models to be bounded. Furthermore, it was shown that these bounds could guide the DM in selecting a more accurate model given a particular design scenario.

5.2 Research Contributions

This section concisely summarizes the major research contributions of this thesis:

1. Information economics was suggested to guide decisions about the formulation of simulation-based design problems.
2. An information economic approach was developed for statistical data collection in support of engineering design decisions.
    a. The information economic principles of Lawrence (Lawrence 1999) were modified such that they could be applied to the case of imprecise knowledge about probabilities.
    b. Using these modified principles, bounds on the value of future statistical data were predicted using imprecise probabilities represented as P-boxes.
    c. The information economic approach to statistical data collections was explored with a pressure vessel design.

3. An information economic approach was developed for model selection in support of engineering design decisions.
    a. Bounds on the value of more accurate models were derived for four cases based on varying assumptions about the level of knowledge of the model's accuracy characterization, the model's cost, the DM's utility function, and the selected decision policy under imprecision.
    b. This value bounding approach was augmented with information economic principles to form an approach for model selection in engineering design.
    c. The approach for model selection was explored with an I-beam structural design problem.

5.3 Limitations

During the completion of this thesis, several limitations of the approaches were discovered. The major limitation is that the approaches presented are computationally burdensome, especially the value bounding approach for statistical data collection presented in Chapter 3. Additionally, we assumed that if the information does not change the DM's decision, than the information has zero value, an assumption that may be limiting for future extensions of this work. This section describes these two limitations such that they can be considered in the further development of the proposed approaches.

5.3.1 Computational cost

The information economic approach for statistical data collection that was presented in Chapter 3 requires a double-loop Monte Carlo simulation for every sample size. For the example problem, the calculation of bounds on the value of the next sample takes about 5 minutes with a high number of p-box and message samples, though results for runs as short as 30 seconds appear nearly as good. These times are on a single 2.6 GHz Pentium 4 processor system with 512 MB of RAM. Although this computation time seems perfectly reasonable, the computational complexity can be expected to increase substantially for more complicated design problems; hence, we recognize that the proposed approach needs to be modified for application to complex design problems. For example, some p-box computations can be performed using algorithms with foundations in interval analysis that do not require second order Monte Carlo techniques (Williamson and Downs 1990; Ferson 2002), and are consequently much less computationally expensive on average (Ferson and Ginzburg 1996). Additionally, new sampling methods have been recently derived for propagating p-boxes through black box models (Bruns 2006; Bruns and Paredis 2006; Bruns, Paredis et al. 2006). Future work investigating how to adapt these methods for computing and simulating directly with p-boxes such that they can be used in the proposed approach is needed.

The information economic approach for model selection presented in Chapter 4 is less computationally expensive than the approach for statistical data collection, but it still may be computationally burdensome for complex design problems. This approach requires an optimization to solve the original design decision, an optimization to find the maximum possible utility, $\overline{U}_{\max}$, and an optimization to find the lower bound on value of each more accurate model being considered. For the experiment presented in Chapter 4, the computation time is negligible, but the computational complexity is expected to increase substantially for more complicated design problems. Whether such computational cost limit the applicability of the approach has not been explored.

5.3.2 The decision unchanged equates to zero value?

When the information economic principles were modified to handle imprecision, we assumed that the value of the new information could be measured directly by noting how the information changed the design decision. Specifically, the value of information is equal to the change in payoff that the DM received based on the new decision. A special case occurs when the DM makes the same decision after the additional information is incorporated into his or her knowledge. According to our assumptions, the value of this additional information is zero, since the DM achieves the same utility from the design decision before and after receiving the information. Although, we assume that this case yields zero value, it should be noted that the possible payoff interval is reduced in size. Such a reduction would often lead to greater DM confidence and therefore may have value in that it could change the DM's behavior in subsequent design decisions. Changing the way the value of information is measured may impact the effectiveness of the approaches proposed, but such an investigation is beyond the scope of this thesis.

5.4 Potential extensions

This thesis lays a foundation for applying information economics to engineering design decisions involving statistical data collection and model selection, but there is still significant room for improvement and additional exploration. This section explores some of the more promising avenues for future work. It is hoped that with future work the approaches presented in this chapter can be applied to a broader class of problems.

5.4.1 Decision making under imprecision

Additional investigation regarding the selection of decision policies under imprecision is needed, both pertaining to the information management decision and the design decision. Such investigation could lead to a systematic framework for selecting decision policies or heuristic knowledge about which decision policies should be applied to particular problem classes.

5.4.1.1 Decision policies for gathering information

In Chapter 3 and Chapter 4, we have presented an approach for bounding the value of collecting additional data samples and bounding the value of more accurate models, respectively. These value bounds need to be resolved according to some policy in order to make a decision. Given just the bounds, any policy that selects a point between the bounds is rational, because the true value is only known to be somewhere between them.

For certain problems, a particular decision policy may be preferable. For example, in Chapter 3, we loosely compare the maximax and midpoint policies for the pressure vessel design example and find that the midpoint policy almost always performs better. If such results could be generalized to specific sets of problems, then designers might be able to choose an appropriate decision policy based on meta-information about the design problem. This would greatly increase the impact of the approach for bounding the value of information presented in this thesis. Whether such generalizations are possible is an open research question.

5.4.1.2 Design decision policies

As mentioned in the "Estimating the value of information" section in Chapter 3, we chose a design decision policy based on precise probabilities (the best-fit normal distribution) so that we could focus on the effectiveness of using imprecise probabilities to represent the DM's state of information and using this imprecise representation to estimate the value of information. Previous work has shown the value of incorporating the imprecision in the DM's state of information directly into the design decision policy (Aughenbaugh and Paredis 2005). The use of imprecise probabilities for both the design decision policy and the prediction of the value of information appears to be a more realistic representation of a typical design problem and could possibly lead to additional insight.

The model selection approach presented in Chapter 4 is based on a maxi-min decision policy for the design decision. Limiting the work to this decision policy allows for simplifications in the approach and computational efficiencies; however, the DM may not wish to use such a conservative decision policy. Generalizing the approach for model selection to other common decision policies would be useful.

5.4.2 Decision problems

The pressure vessel and the I-beam example used in this thesis are deliberately simple in order to clearly illustrate the application of the proposed approaches. To assess the general applicability of the approaches, the design examples need to be varied. Specifically, several variations of the decision problems could be investigated. First, a design problem in which there are multiple uncertain parameters and multiple sources of information should be explored. Such an extension may lead to determining not only what model to develop, but also what uncertainties impact the decision most. Second, a decision problem involving both irreducible and reducible uncertainty could be considered to investigate how the level of irreducibility impacts the information management decisions. Finally, the effects of varying levels of risk-preference could be explored. Each possible extension would lead to more general conclusions about the applicability of the approaches presented in this thesis.

5.4.3 P-box construction

When constructing p-boxes, it would be impractical to use 100% confidence intervals, since these would be infinite. For the example problem in Chapter 3, we construct p-boxes using 95% confidence intervals on the mean and variance and assume that they contain the truth (Aughenbaugh, Ling et al. 2005). It is possible to create p-boxes based on weaker assumptions using alternative methods for p-box construction (Ferson 2002). Similar to the decision policy work noted above, different p-box construction policies may work better for different design problems. Any such relationships need to be explored before this approach can be put to practical use.

5.4.4 Unknown distribution types

The p-box formalism can be used to represent the DM's lack of knowledge about a distribution when the DM has varying amounts of initial knowledge (Ferson 2002). In Chapter 3, we investigated how our approach performs when the DM knows the type of distribution but has no knowledge about the values of the distribution's parameters. Other possible cases include when the DM can specify a set of all possible distribution types or when the DM lacks any knowledge about the possible distribution types. In both cases, the p-box would be broader resulting in wider value bounds. The applicability of our approach could be expanded to other classes of problems by investigating how it performs under varying amounts of initial knowledge about the distribution being characterized.

5.4.5 Considering model dependence

The value bounds that were derived in Chapter 4 are often overly conservative because the models are assumed nondependent (Ferson and Kreinovich 2006). If a coarse model and a more accurate model are nondependent, nothing can be said about the output of the more accurate model expect that it will be a subset of the output for the coarse model. Whereas, models that smoothly converge to an exact prediction would be dependent. In some cases, assuming nondependence among the models is justified; however, in many engineering problems the models are dependent and accounting for this dependence could possibly lead to better model selection decisions. An method for accounting for interval dependencies has been recently introduced (Ferson and Kreinovich 2006). Integrating such a method into the model selection approach could prove useful.

5.5 Closing Statement

Thinking of design at the meta-level, i.e., designing the design process, has great potential for improving the design of complex systems. In this thesis, we have incorporated information management decisions into the design decision framework. In doing so, we have discovered that there is significant potential for applying information economics in the design process. As more researchers and practicing engineers recognize the value of using information economic principles in the design process, we hope that the results presented in this thesis can be extended to improve the design of complex engineering systems.

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